10

СТАТИКА КОНСТРУКЦИЈА 1

Модул: Конструкције

– материјал за вежбе –

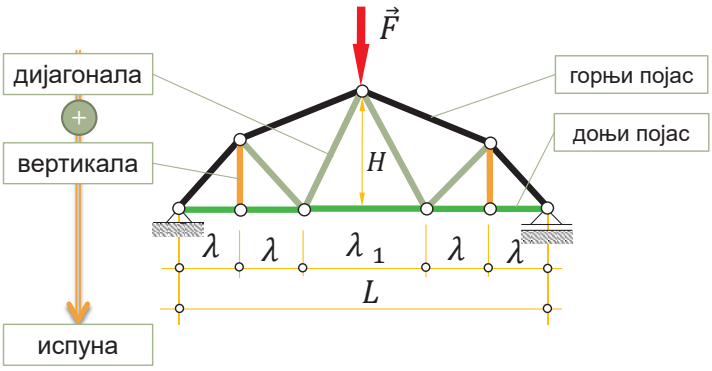
2024.

Решеткасти носачи у равни

Основне карактеристике су:

- код решетке имамо само праве зглавкасто везане штапове,
- у чвору осе свих штапова морају да се секу у једној тачки,
- активне силе делују само у чворовима решетке,
- ослонци су само у чоровима.

Елементи решеткастог носача



Обележавање:

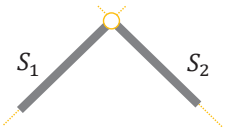
- k - број чворова решетке,

Прорачун непознатих сила у штаповима:

- Метода чворова,
- Метода пресека,
- Аналитички изрази за силе у штаповима (и њихова примена за конструисање утицајних линија),

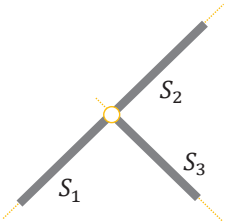
Метода чворова – „ шест случајева “

не оптерећен чвор



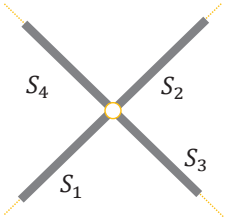
$S_1 = 0$

$S_2 = 0$



$S_1 = S_2$

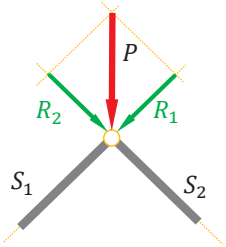
$S_3 = 0$



$S_1 = S_2$

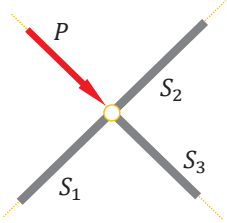
$S_3 = S_4$

оптерећен чвор



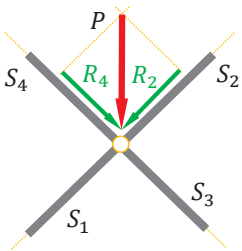
$S_1 = R_1$

$S_2 = R_2$



$S_1 = S_2$

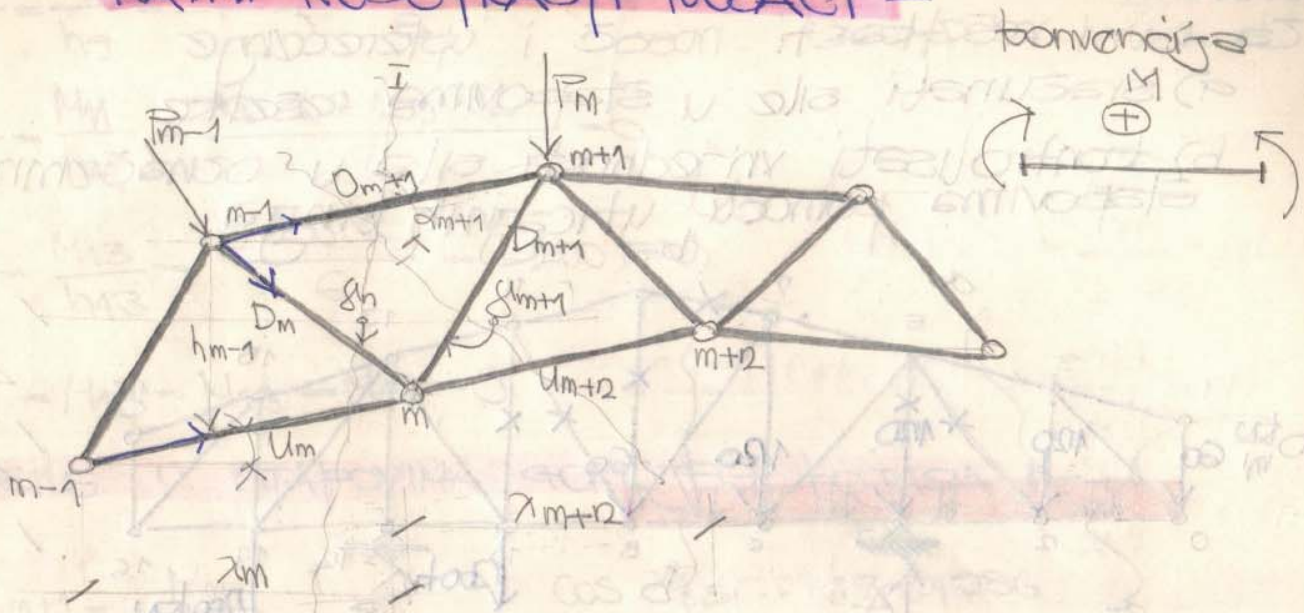
$S_3 = P$



$S_1 + S_2 = R_2$

$S_3 + S_4 = R_4$

– RAVNI REŠETKASTI NOSAČI –



Analitički izrazi za sile u štapovima sa trouganom ispunom

$$\Sigma M_m^{I-I} = 0 \Rightarrow M_m + O_{m+1} \cdot \cos\alpha_{m+1} \cdot h_m = 0$$

$$O_{m+1} = -\frac{M_m}{P_m} \cdot \frac{1}{\cos\alpha_{m+1}}$$

$$\Sigma M_{m-1}^{I-I} = 0 \Rightarrow M_{m-1} - U_m \cdot \cos\beta_m \cdot h_{m-1}$$

$$U_m = \frac{M_{m-1}}{h_{m-1}} \cdot \frac{1}{\cos\beta_m}$$

$$\Sigma H^{I-I} = 0 \Rightarrow O_{m+1}\cos\alpha_{m+1} + D_m \cdot \cos\beta_m \cdot \cos\delta_m + U_m \cdot \cos\beta_m + H_m = 0$$

$$D_m = \left(\frac{M_m}{h_m} - \frac{M_{m-1}}{h_{m-1}} - H_m\right) \cdot \frac{1}{\cos\delta_m}$$

dijagonala pada s lijeva na desno

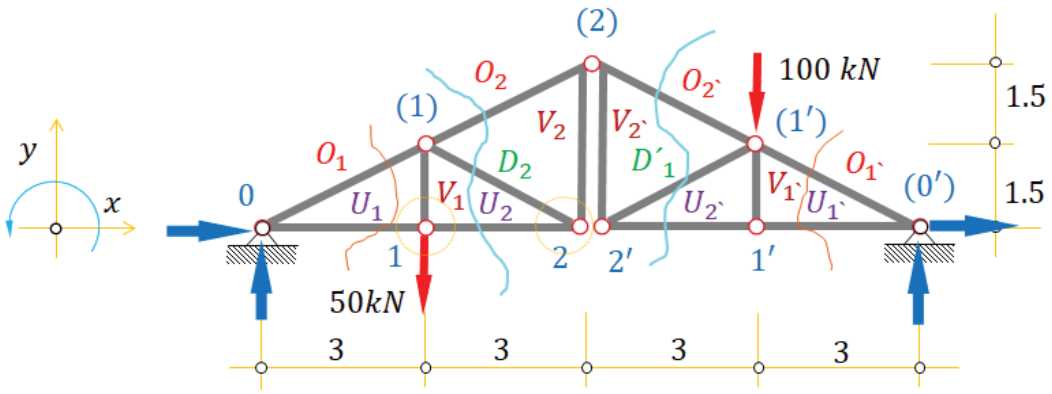
$$\Sigma H^{II-II} = 0 \Rightarrow O_{m+1} \cdot \cos\alpha_{m+1} + D_{m+1} \cdot \cos\delta_{m+1} + U_{m+2} \cdot \cos\beta_{m+2} + H_{m+2} = 0$$

$$D_{m+1} = \left(\frac{M_m}{h_m} - \frac{M_{m+1}}{h_{m+1}} - H_{m+2}\right) \cdot \frac{1}{\cos\delta_{m+1}}$$

dijagonala pada s desna na lijevo

Пример

За носач са датим оптерећењем приказ на скици срачунати силе у штаповима на основу аналитичких израза.



Реакције веза:

Given $P_1 := 50\ kN$ $P_2 := 100\ kN$

$\Sigma H = 0$ $H_a + H_b = 0$

$\Sigma V = 0$ $V_a + V_b - P_1 - P_2 = 0$

$\Sigma M_a = 0$ $V_b \cdot 12 - P_1 \cdot 3 - P_2 \cdot 9 = 0$

$\Sigma M_{2D} = 0$ $V_b \cdot 6 - H_b \cdot 3 - P_2 \cdot 3 = 0$

$$\text{Find}(H_a, H_b, V_a, V_b) \rightarrow \begin{pmatrix} -75 \\ 75 \\ \frac{125}{2} \\ \frac{175}{2} \end{pmatrix}$$

$H_a := -75\ kN$ $H_b := 75\ kN$ $V_a := \frac{125}{2} = 62.5\ kN$ $V_b := \frac{175}{2} = 87.5\ kN$

Моменти савијања у чворовима решеткастог носача:

$M_1 := V_a \cdot 3$ $M_1 = 187.5$

$M_{1G} := V_a \cdot 3 + H_a \cdot 1.5$ $M_{1G} = 75$

$M_2 := V_a \cdot 6 - P_1 \cdot 3$ $M_2 = 225$

$M_{2G} := V_a \cdot 6 - P_1 \cdot 3 + H_a \cdot 3$ $M_{2G} = 0$

$M_{2'} := V_b \cdot 6 - P_2 \cdot 3$ $M_{2'} = 225$

$M_{1'} := V_b \cdot 3$ $M_{1'} = 262.5$

$M_{1'G} := V_b \cdot 3 - H_b \cdot 1.5$ $M_{1'G} = 150$

Силе у штаповима на основу аналитичких израза

$\alpha := 26.57deg$ $\beta := 0deg$ $h_1 := 1.5$ $h_2 := 3$

$U_2 := \frac{M_{1G}}{h_1} \cdot \frac{1}{\cos(\beta)}$ $U_2 = 50 \ \ kN$ $U_1 = U_2$

$U_{1'} := \frac{M_{1'G}}{h_1} \cdot \frac{1}{\cos(\beta)}$ $U_{1'} = 100 \ \ kN$ $U_{1'} = U_{2'}$

$O_2 := \frac{-M_2}{h_2} \cdot \frac{1}{\cos(\alpha)}$ $O_2 = -83.86 \ \ kN$

$O_1 := \frac{-M_1}{h_1} \cdot \frac{1}{\cos(\alpha)}$ $O_1 = -139.76 \ \ kN$

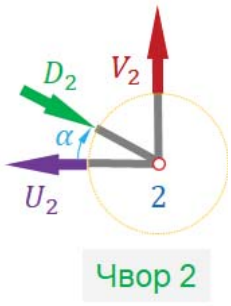
$O_{2'G} := \frac{-M_{2'}}{h_2} \cdot \frac{1}{\cos(\alpha)}$ $O_{2'G} = -83.86 \ \ kN$

$O_{1'G} := \frac{-M_{1'}}{h_1} \cdot \frac{1}{\cos(\alpha)}$ $O_{1'G} = -195.66 \ \ kN$

$$D_2 := \left(\frac{M_2}{h_2} - \frac{M_{1G}}{h_1} + H_a\right)\cdot\frac{1}{\cos(\alpha)} \qquad D_2 = -55.9 \text{ kN}$$

$$D_{1'} := \left(\frac{M_{2'}}{h_2} - \frac{M_{1'G}}{h_1} - H_b\right)\cdot\frac{1}{\cos(\alpha)} \qquad D_{1'} = -111.81 \text{ kN}$$

чвор 1 $\Sigma V = 0$ Given $V_1 - 50 = 0$ $\text{Find}(V_1) \rightarrow 50$ kN

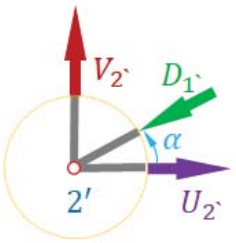


Чвор 2

чвор 2 $\Sigma V = 0$ Given $V_2 - D_2\cdot\sin(\alpha) = 0$

$$\text{Find}(V_2) \text{ simplify } \rightarrow -\frac{50.0\cdot\sin(26.57\cdot\text{deg})}{\cos(26.57\cdot\text{deg})^{1.0}}$$

$$\frac{50.0\cdot\sin(26.57\cdot\text{deg})}{\cos(26.57\cdot\text{deg})^{1.0}} = 25.01 \text{ kN}$$

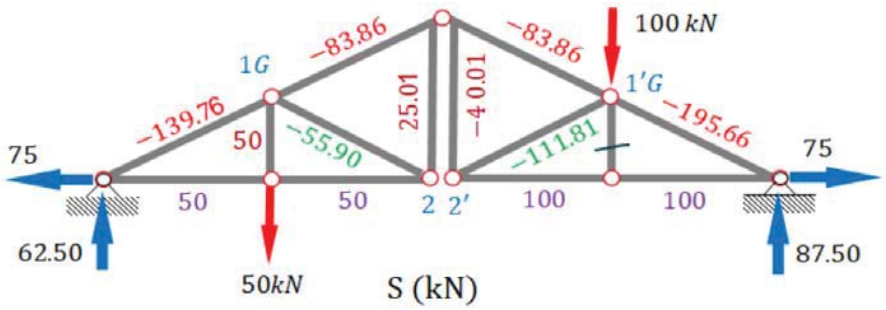


Чвор 2`

чвор 2` $\Sigma V = 0$ Given $V_{2'} - D_{1'}\cdot\sin(\alpha) = 0$

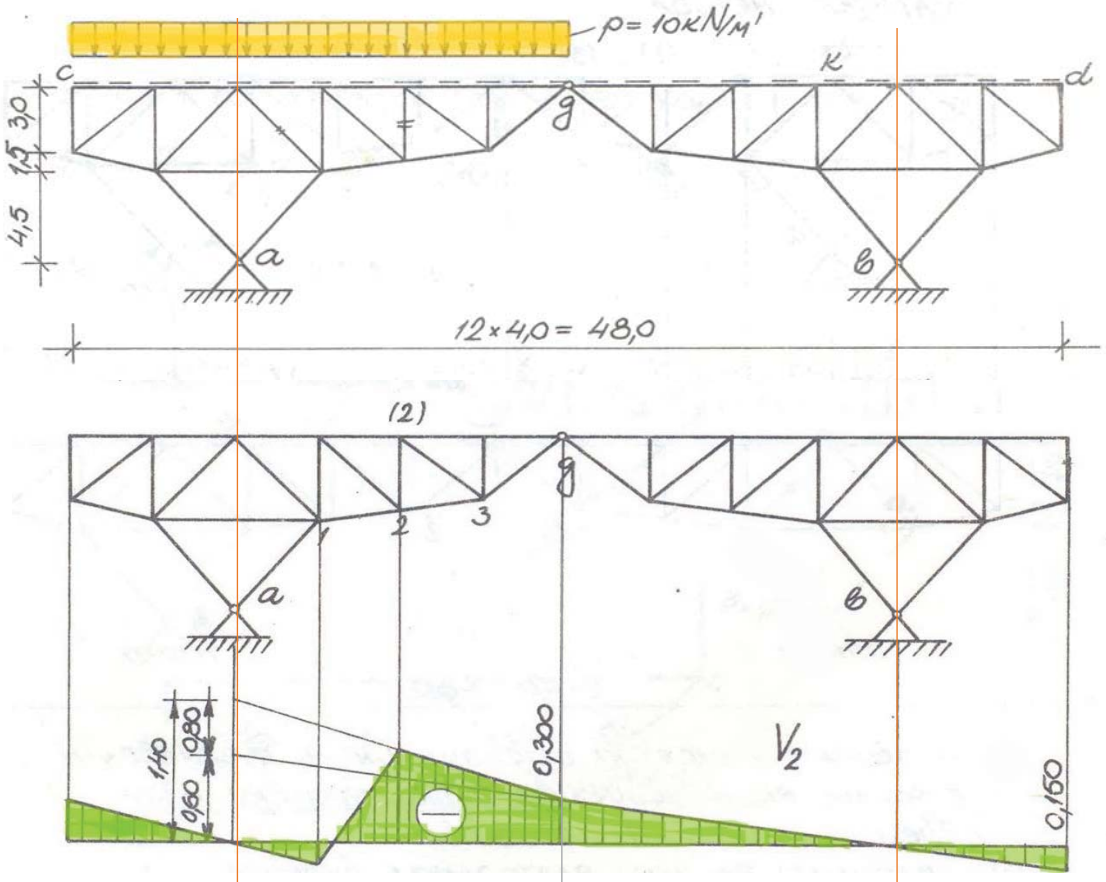
$$\text{Find}(V_{2'}) \text{ simplify } \rightarrow -\frac{100.0\cdot\sin(26.57\cdot\text{deg})}{\cos(26.57\cdot\text{deg})^{1.0}}$$

$$-\frac{80.0\cdot\sin(26.57\cdot\text{deg})}{\cos(26.57\cdot\text{deg})^{1.0}} = -40.01 \text{ kN}$$



Пример

За дати носача приказан на скици услед задатог оптерећења срачунати силу у означеном вертикалном штапу.



Аналитички изрази су:

$$V_m = -\frac{h_{m+1}}{\lambda}\left(\frac{M_{m+1}}{h_{m+1}} - \frac{M_m}{h_m}\,\frac{d_{m+1}}{h_{m+1}}\right) - P_m$$

$$V_m = -\frac{h_{m+1}}{\lambda}\left(\frac{M_{m+1}}{h_{m+1}} - \frac{M_m}{h_m}\right) - P_m$$

односно, преко одсечка над ослонцем „а“:

$$V_{mA} = -\frac{h_3}{\lambda}\left(\frac{x_3}{h_3} - \frac{x_2}{h_2}\right) = -\frac{3,0}{4,0}\left(\frac{3\cdot 4,0}{3,0} - \frac{2\cdot 4,0}{3,75}\right) = -1,40$$

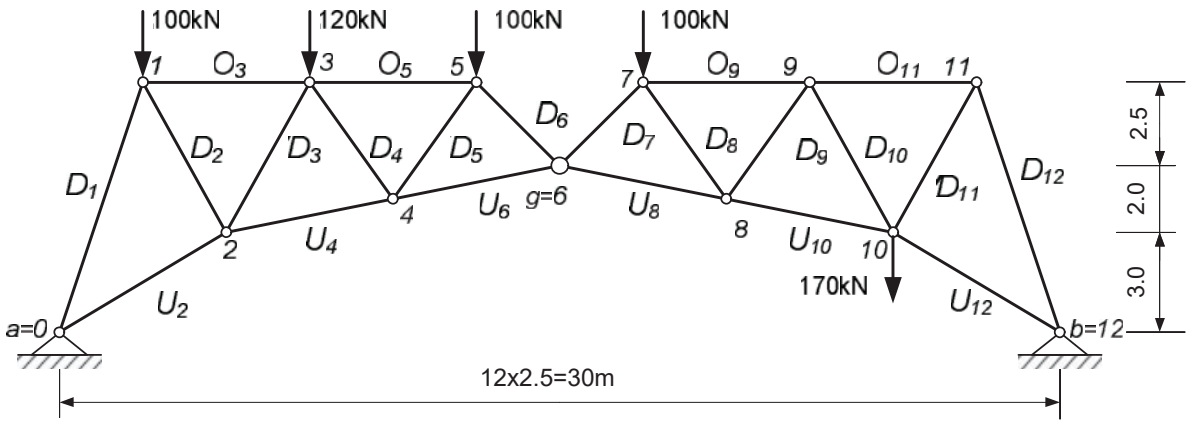
$$V_{mH} = \frac{h_3}{\lambda}\left(\frac{y_3}{h_3} - \frac{y_2}{h_2}\right) = \frac{3,0}{4,0}\left(\frac{6,0}{3,0} - \frac{5,25}{3,75}\right) = 0,45$$

$$\frac{l_2}{f}V_{2H} = \frac{16,0}{9,0} \times 0,45 = 0,80$$

$$V_{mB} = -\frac{3,0}{4,0}\left(\frac{5\cdot 4,0}{3,0} - \frac{6\cdot 4,0}{3,75}\right) = -0,20$$

Пример

За носач са датим оптерећењем приказ на скици срачунати силе у штаповима на основу аналитичких израза.



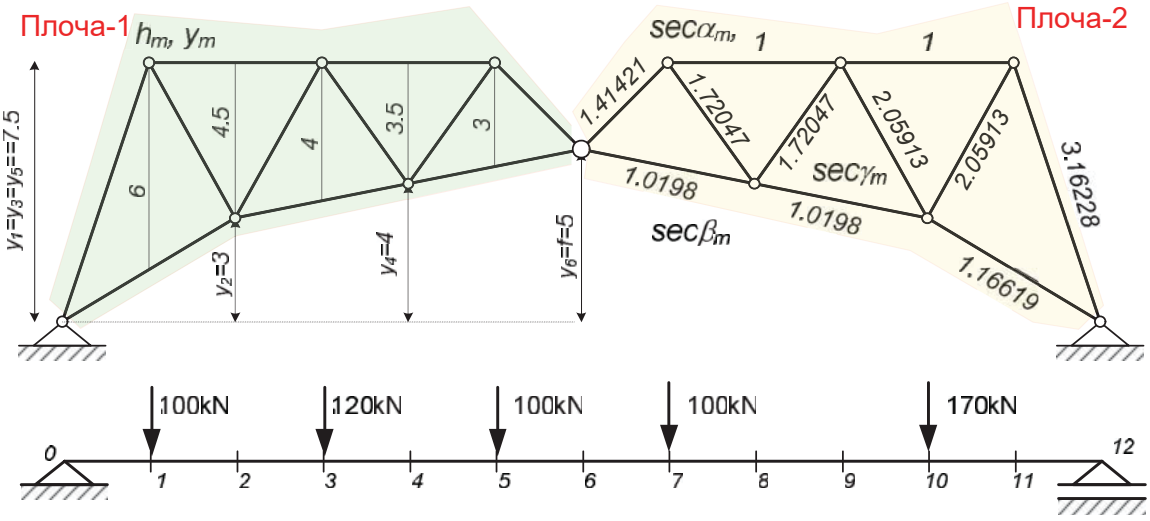
Аналитички изрази за силе у штаповима решеткастог носача.

за.....m=3,5,9,11 $$O_m = -\frac{M_{m-1}}{h_{m-1}} \cdot \sec\alpha_m$$

за....m=2,4,6,8,10,12 $$U_m = \frac{M_{m-1}}{h_{m-1}} \cdot \sec\beta_m$$

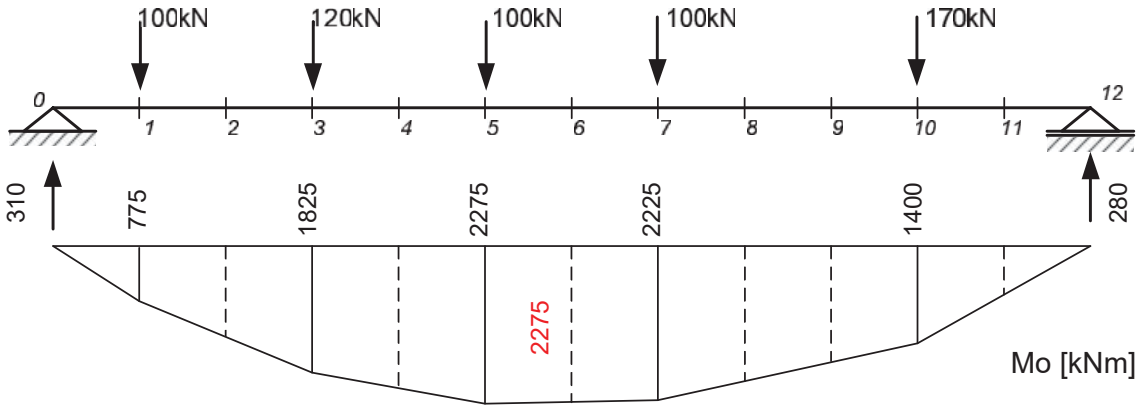
за.....m=1,3,5,7,9,11 $$D_m = \left(\frac{M_{m-1}}{h_{m-1}} - \frac{M_m}{h_m} - H_m\right) \cdot \sec\gamma_m$$

за....m=2,4,6,8,10,12 $$D_m = \left(\frac{M_m}{h_m} - \frac{M_{m-1}}{h_{m-1}} - H_m\right) \cdot \sec\gamma_m$$



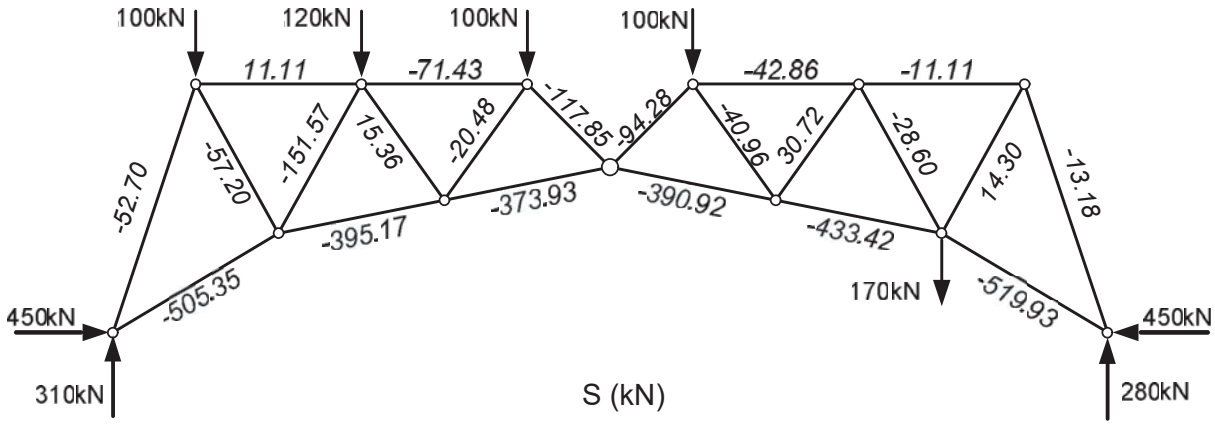
$M_m = M_{m0} - H \cdot y_m$

M_{m0} - моменти савијања одговарајуће просте греде

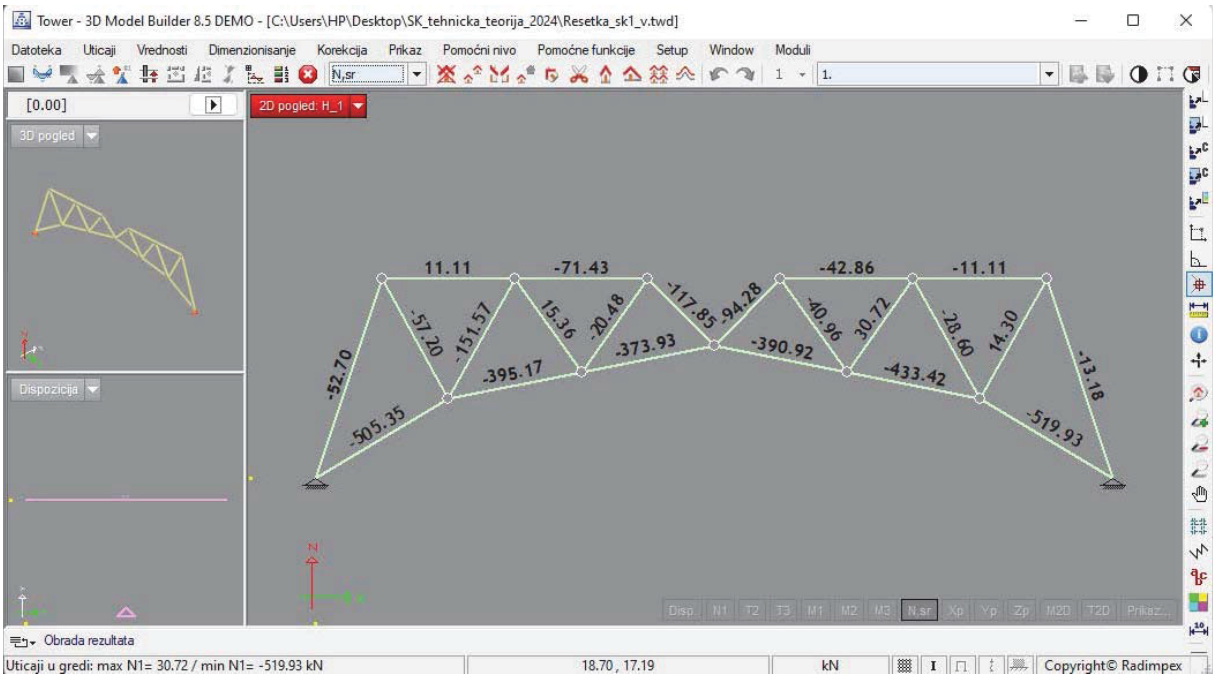


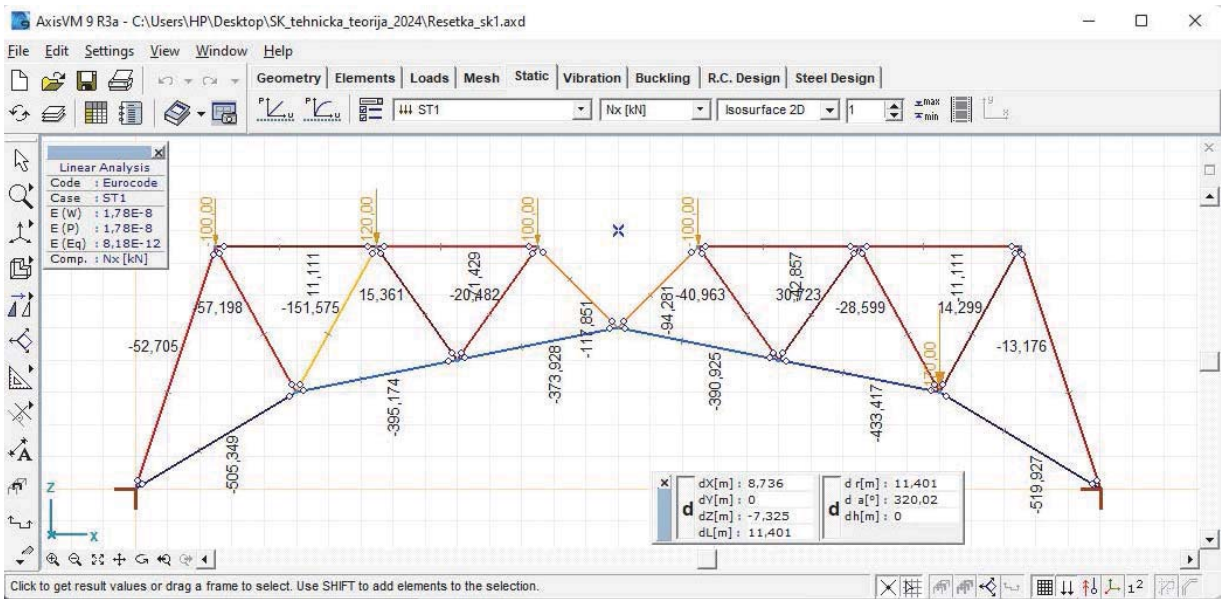
$H = M_{g0}/f = 2250/5 = 450$ kN

m	M_{mo}	$H \cdot y_m$	M_m	M_m/h_m	U_m	O_m	D_m
0	0	0	0	0	-	-	-
1	775	3375	-2600	-433.33	-	-	-52.70
2	1300	1350	-50	-11.11	-505.35	-	-57.20
3	1825	3375	-1550	-387.5	-	11.11	-151.57
4	2050	1800	250	71.43	-395.17	-	15.36
5	2275	3375	-1100	-366.67	-	-71.43	-20.48
6	2250	2250	0	0	-373.93	-	-117.85
7	2225	3375	-1150	-383.33	-	-	-94.28
8	1950	1800	150	42.86	-390.92	-	-40.96
9	1675	3375	-1700	-425.00	-	-42.86	30.72
10	1400	1350	50	11.11	-433.42	-	-28.60
11	700	3375	-2675	-445.83	-	-11.11	14.30
12	0	0	0	0	-519.93	-	-13.18



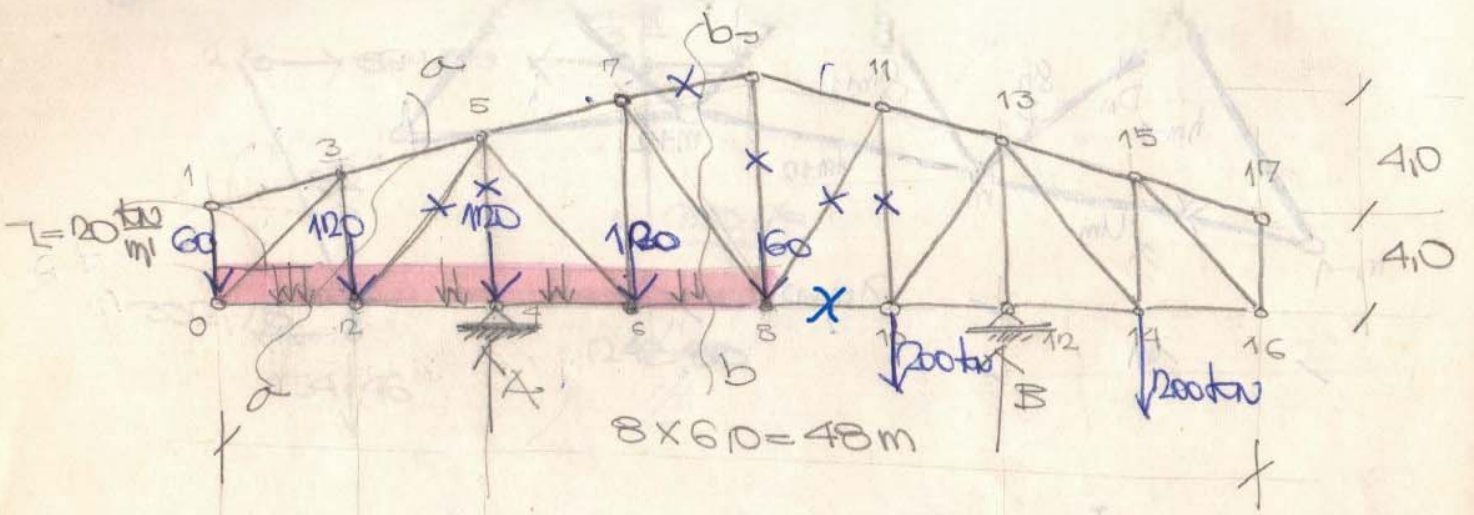
Контрола резултата методом коначних елемената

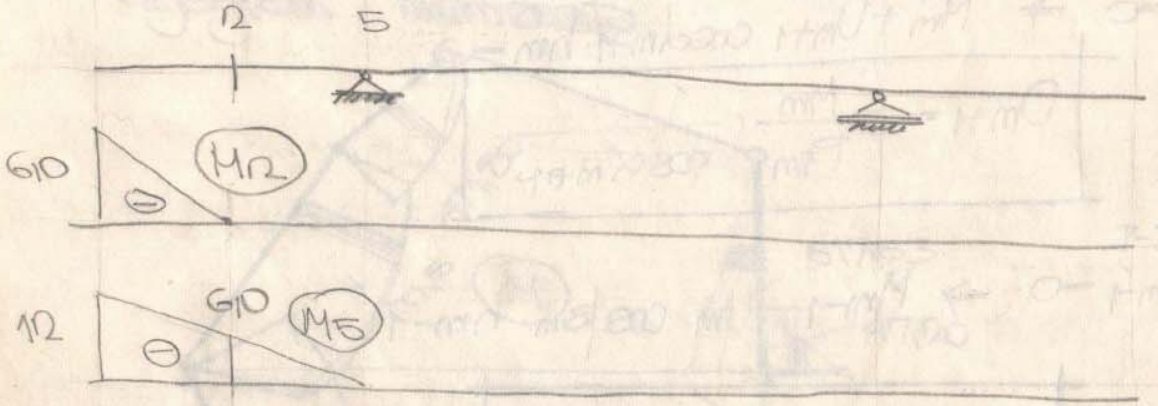


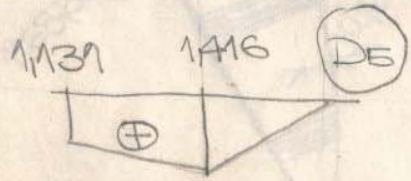


ZADATAK

Za ravni rešetkasti nosač i opterećenje sračunati sile u štapovima rešetke







$D_5 = 60 \cdot 1{,}131 + 120 \cdot 1{,}416 = 237{,}70\ kN$

$\Sigma M_{1,12} = 0 \Rightarrow A = \frac{1}{24}\left[60(36+12) + 120(30+24+18) + 200(6-6)\right] =$

$A = 480\ kN$

$\Sigma V = 0 \qquad B = 2 \cdot 60 + 3 \cdot 120 + 2 \cdot 200 - 480 = 400\ kN$

$\frac{M_{12}}{h_{12}} = \frac{M_3}{h_3} = \frac{-60 \cdot 6}{5} = -72\ kN$

$\frac{M_4}{h_4} = \frac{M_5}{h_5} = \frac{-60 \cdot 12 - 120 \cdot 6}{6} = -240\ kN$

$\frac{M_6}{h_6} = \frac{M_7}{h_7} = \frac{-60 \cdot 18 - 120(12+6) + 480 \cdot 6}{7} = 51{,}43\ kN$

$$\frac{M_8}{h_8} = \frac{M_9}{h_9} = \frac{400 \cdot 12 - 200(6+18)}{8} = 0$$

$$\frac{M_{10}}{h_{10}} = \frac{M_{11}}{h_{11}} = \frac{400 \cdot 6 - 200 \cdot 12}{7} = 0$$

$$\frac{M_{12}}{h_{12}} = \frac{M_{13}}{h_{13}} = \frac{-200 \cdot 6}{6} = -200\ kN$$

$$M_{14} = M_{15} = M_{16} = M_{17} = 0$$

+ SILE U STAPOVIMA GORNJEG POJASA +

$$O_{m+1} = -\frac{M_m}{h_m} \cdot \frac{1}{\cos\alpha_{m+1}} \qquad \cos\alpha_{3,5\ldots17} = 0{,}986$$

$$O_3 = -\frac{M_0^{\,0}}{h_0} \cdot \frac{1}{\cos\alpha_3} = 0$$

$$O_5 = -\frac{M_2}{h_2} \cdot \frac{1}{\cos\alpha_5} = -(-72{,}0) \cdot \frac{1}{0{,}986} = 73{,}02\ kN$$

$$O_7 = -\frac{M_6}{h_6} \frac{1}{\cos\alpha_7} = -(51{,}43) \cdot \frac{1}{0{,}986} = 52{,}16\ kN$$

$$O_9 = -\frac{M_8^{\,0}}{h_8} \cdot \frac{1}{\cos\alpha_9} = 0$$

$$O_{11} = -\frac{M_8^{\,0}}{h_8} \cdot \frac{1}{\cos\alpha_{11}} = 0$$

└ Ritarova tačka za štap 11

$$O_{13} = -\frac{M_{10}^{\,0}}{h_{10}} \cdot \frac{1}{\cos\alpha_{13}} = 0$$

$$O_{15} = O_{17} = 0$$

+ SILE U ŠTAPOVIMA DONJEG POJASA +

$$U_m = \frac{M_{m-1}}{h_{m-1}} \cdot \frac{1}{\cos\beta_m} \qquad \cos\beta_{2,4\ldots16} = 1{,}0$$

$$U_2 = \frac{M_3}{h_3} = -72\ kN$$

└ Ritarova tačka za štap 2

$$U_4 = \frac{M_5}{h_5} = -240\ kN$$

$$U_6 = \frac{M_5}{h_5} = -240\ kN$$

$$U_8 = \frac{M_7}{h_7} = -51{,}43\ tN$$

$$U_{10} = \frac{M_{11}}{h_{11}} = 0$$

$$U_{12} = \frac{M_{13}}{h_{13}} = -200\ tN$$

$$U_{14} = \frac{M_{13}}{h_{13}} = -200\ tN$$

$$U_{16} = \frac{M_{15}}{h_{15}} = 0$$

$$Z_S + Z_O = 2t$$

t-broj čvorova
Z_S = sile u štapu
Z_O = reakcija oslonaca

+ SILE U DIJAGONALAMA +

$$\cos\varphi_3 = \cos\varphi_{16} = 0{,}768$$

$$\cos\varphi_5 = \cos\varphi_6 = \cos\varphi_{13} = \cos\varphi_{14} = \frac{\sqrt{2}}{2}$$

$$\cos\varphi_8 = \cos\varphi_{11} = 0{,}651$$

$$\swarrow D_3 = \left(\frac{M_0}{h_0} - \frac{M_3}{h_3} + \cancel{N_2}^{t0}\right) \cdot \frac{1}{\cos\varphi_3} = -(-72{,}0)\frac{1}{0{,}768} = 93{,}75\ tN$$

$$D_5 = \left(\frac{M_2}{h_2} - \frac{M_5}{h_5}\right) \cdot \frac{1}{\cos\varphi_5} = \left(-72{,}0 - (-240)\right) \cdot \frac{2}{\sqrt{2}} = 237{,}76\ tN$$

$$\downarrow D_6 = \left(\frac{M_6}{h_6} - \frac{M_5}{h_5}\right) \frac{1}{\cos\varphi_6} = \left(-51{,}43 - (-240)\right) \frac{2}{\sqrt{2}} = 266{,}72\ tN$$

$$D_8 = \left(\frac{M_8}{h_8} - \frac{M_7}{h_7}\right) \cdot \frac{1}{\cos\varphi_8} = \left(0 - (-51{,}43)\right) \frac{1}{0{,}651} = 79\ tN$$

$$\swarrow D_{11} = \left(\frac{M_8}{h_8} - \frac{M_{11}}{h_{11}}\right) \cdot \frac{1}{\cos\varphi_{11}} = (0 - 0) \cdot \frac{1}{0{,}651} = 0$$

$$D_{13} = \left(\frac{M_{10}}{h_{10}} - \frac{M_{13}}{h_{13}}\right) \frac{1}{\cos\varphi_{13}} = \left(0 - (-200)\right) \frac{2}{\sqrt{2}} = 282{,}89\ tN$$

$$\downarrow D_{14} = \left(\frac{M_{14}}{h_{14}} - \frac{M_{13}}{h_{13}}\right) \cdot \frac{1}{\cos\varphi_{14}} = \left(0 - (-200)\right) \cdot \frac{2}{\sqrt{2}} = 282{,}89\ tN$$

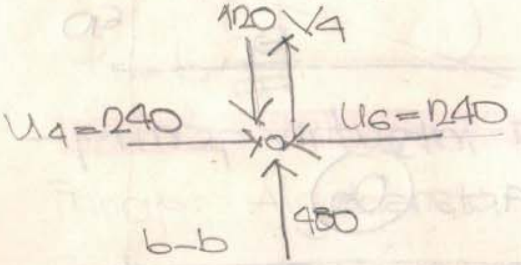
$$D_{16} = 0$$

+ SILE U VERTIKALAMA +

$V_0 = V_{16} = V_{10} = V_8 = V_{14} = 0$

$V_2 + 60 - 05 \cdot \sin\alpha_5 = 0$

$V_2 = 73{,}02 \cdot 0{,}164 - 60 = -48{,}02\ kN$



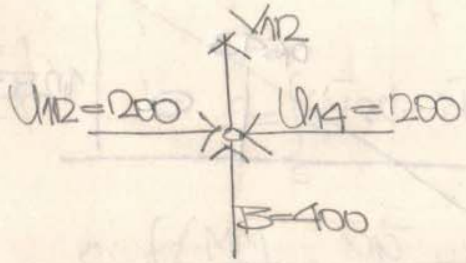
$\Sigma V_A = 0$

$V_4 = -360\ kN$

$\Sigma V_{lijevo} = 0$

$V_6 + 07 \cdot \sin\alpha_7 + 60 = 0$

$V_6 = -52{,}16 \cdot 0{,}164 - 60 = -68{,}55\ kN$



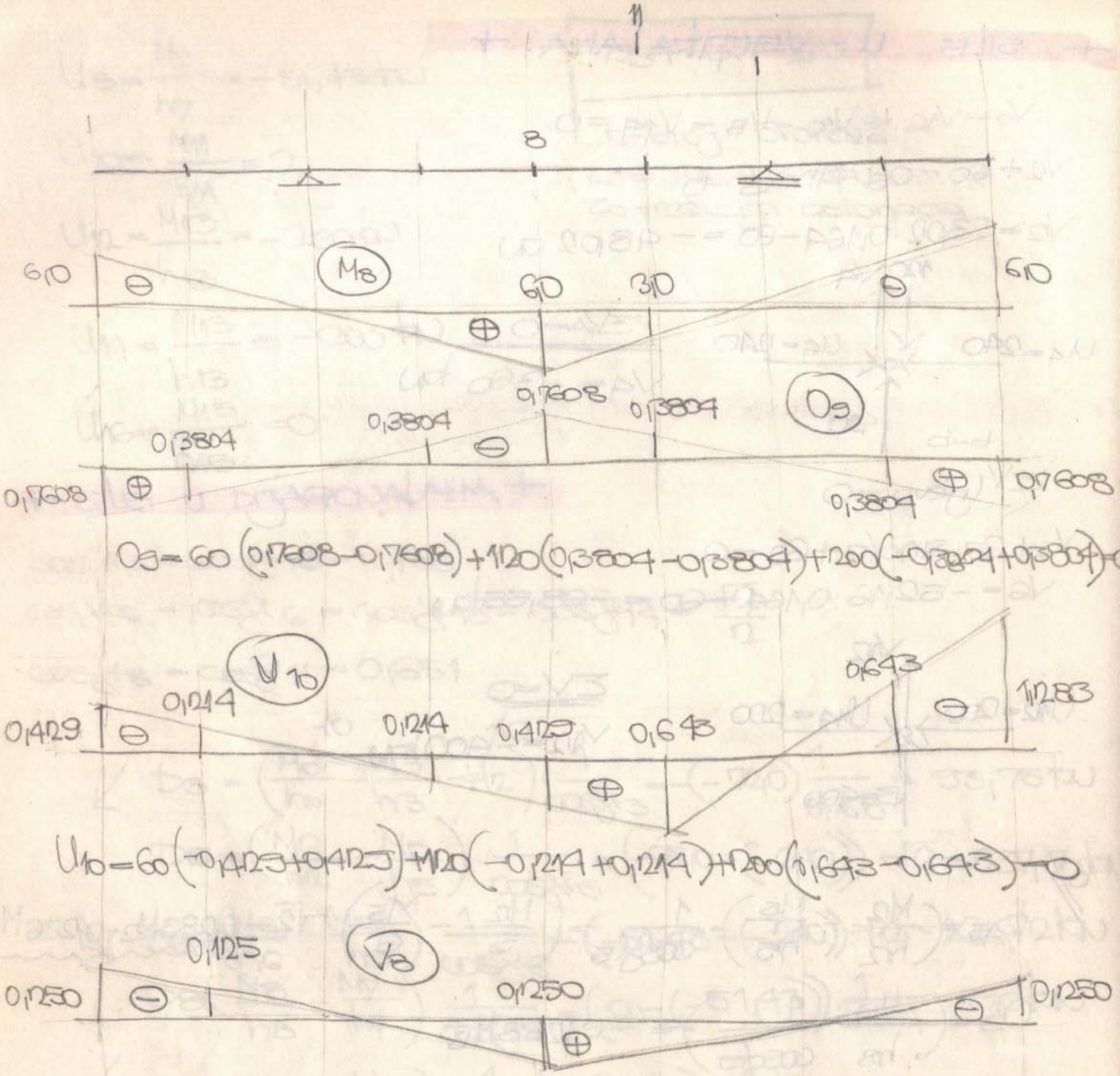
$\Sigma V = 0$

$V_{12} = -400\ kN$

b//

$b_5 = \left(\frac{M_{12}}{h_{12}} - \frac{M_5}{h_5}\right) \cdot \frac{1}{\cos\beta_5} = \left(\frac{M_{12}}{5} - \frac{M_5}{6}\right) \cdot \frac{2}{\sqrt{2}} = 0{,}283 M_{12} - 0{,}236 M_5$

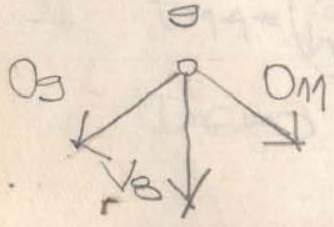
$O_5 = \left(-\frac{M_8}{h_8} \cdot \frac{1}{\cos\alpha_5}\right) = -0{,}1268 M_8$



$$O_3 = 60(0{,}7608 - 0{,}7608) + 120(0{,}3804 - 0{,}3804) + 200(-0{,}3804 + 0{,}3804) = 0$$

$$V_{10} = 60(-0{,}1429 + 0{,}1429) + 120(-0{,}1214 + 0{,}1214) + 200(0{,}643 - 0{,}643) = 0$$

$$V_{10} = \frac{M_{11}}{h_{11}} \cdot \frac{1}{600\beta_{10}} = \frac{M_{11}}{7}$$

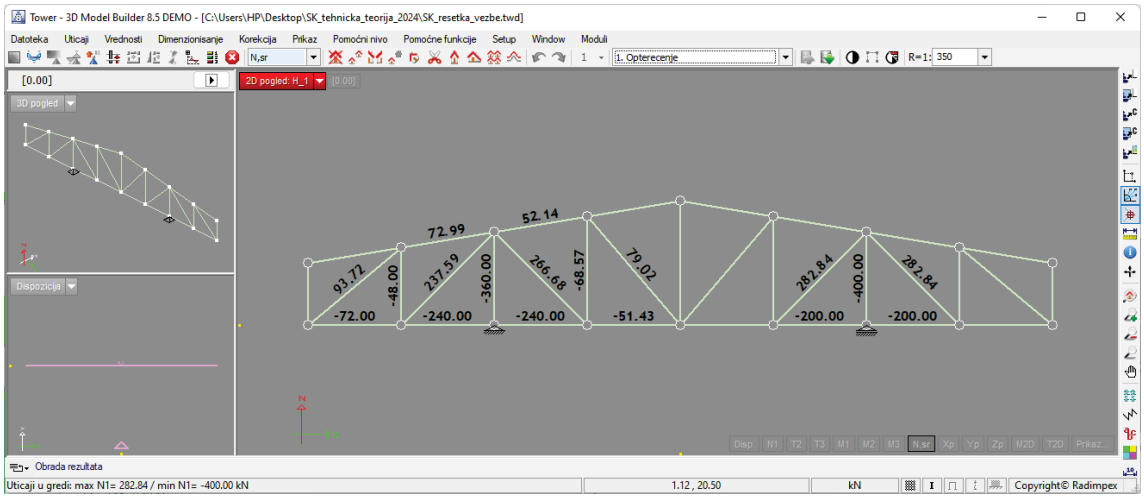


$$\Sigma V_3 = 0$$

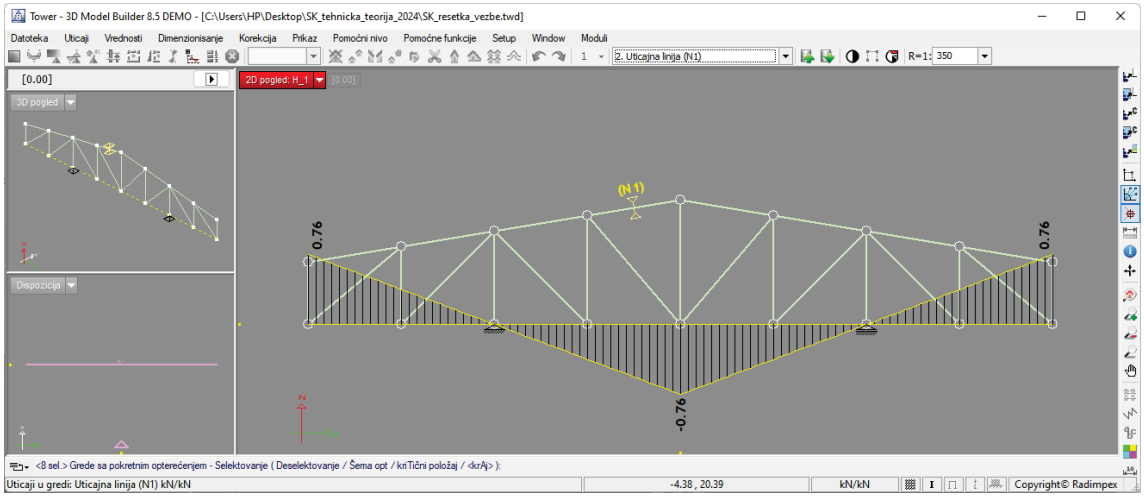
$$O_3 \sin\alpha_3 + O_{11} \cdot \sin\alpha_{11} + V_8 = 0$$

$$V_8 = 2 \cdot \frac{M_8}{h_8} \operatorname{tg}\alpha_{3,11} = 0{,}041 \cdot M_8$$

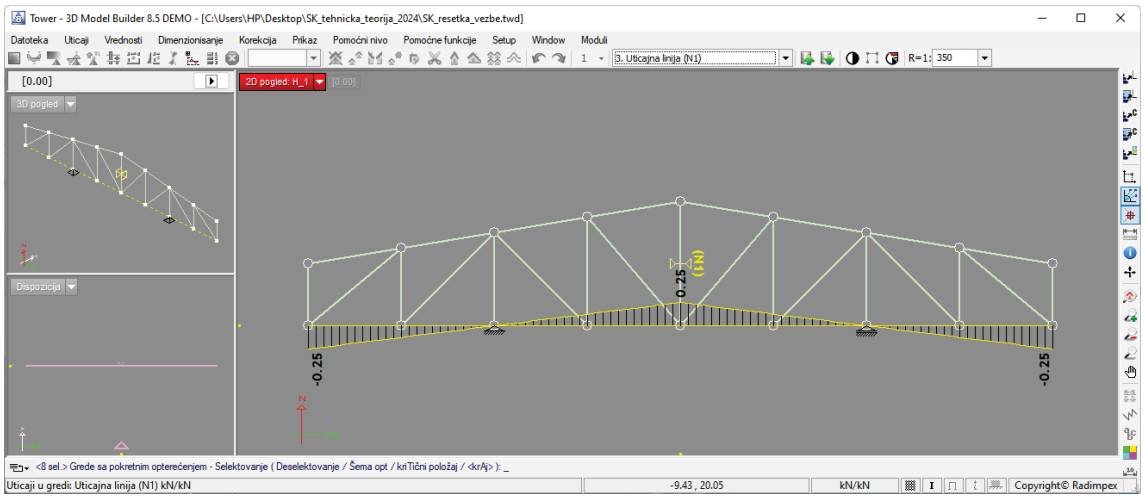
Преглед резултата – метода коначних елемената



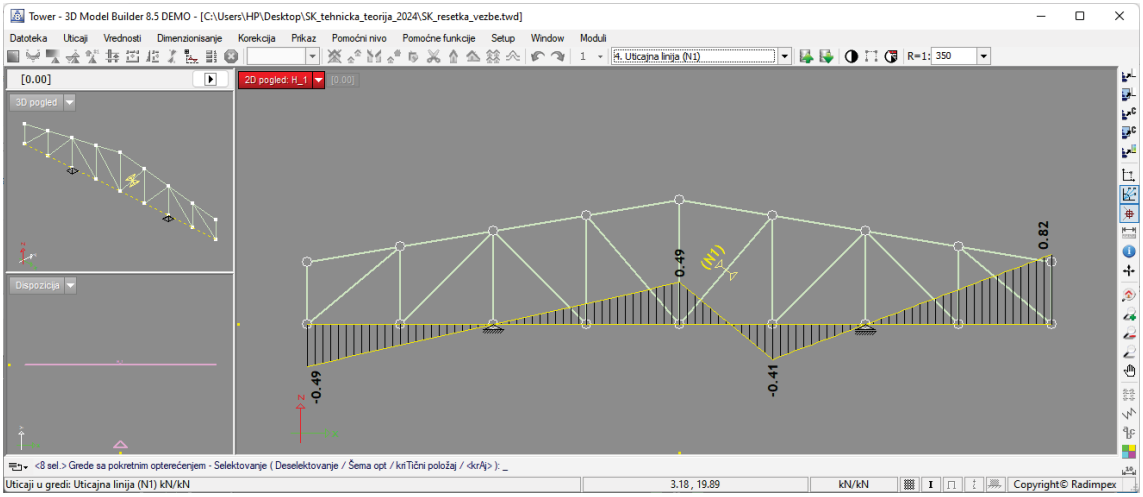
Слика 1: $S(kN)$



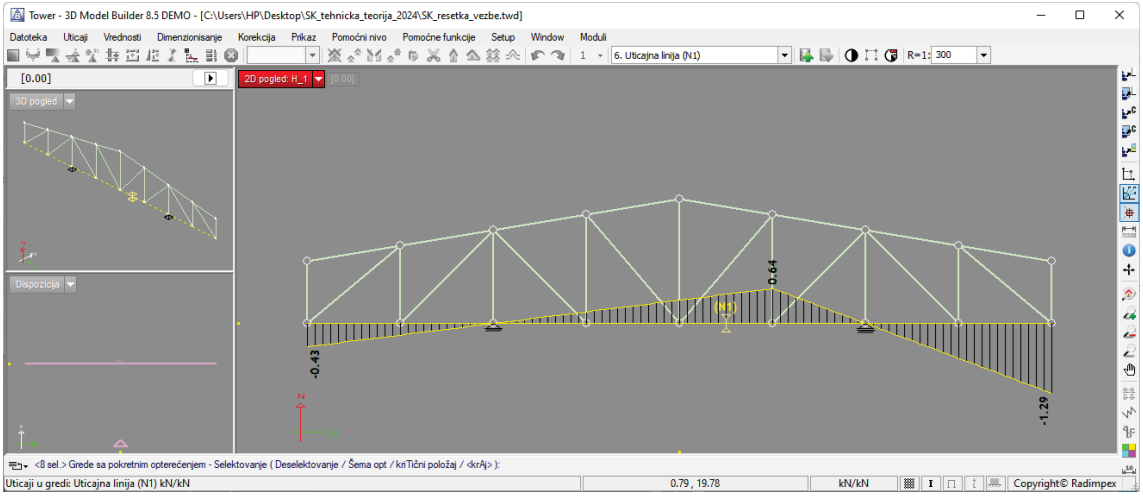
Слика 2: штап O_9 – утицајна линија



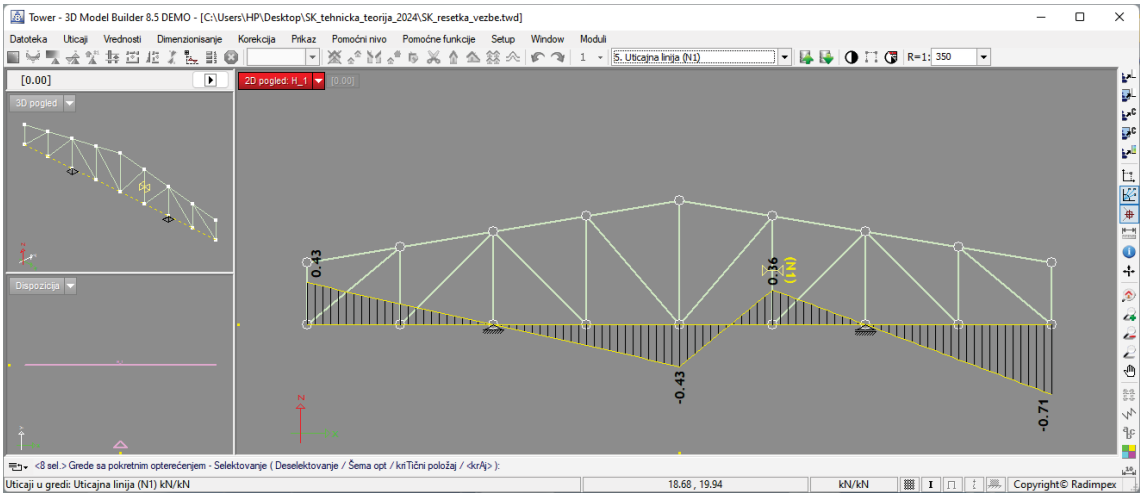
Слика 3: штап V_8 – утицајна линија



Слика 4: штап D_{8-11} – утицајна линија



Слика 5: штап U_{10} – утицајна линија



Слика 6: штап V_{10} – утицајна линија