7

СТАТИКА КОНСТРУКЦИЈА 1

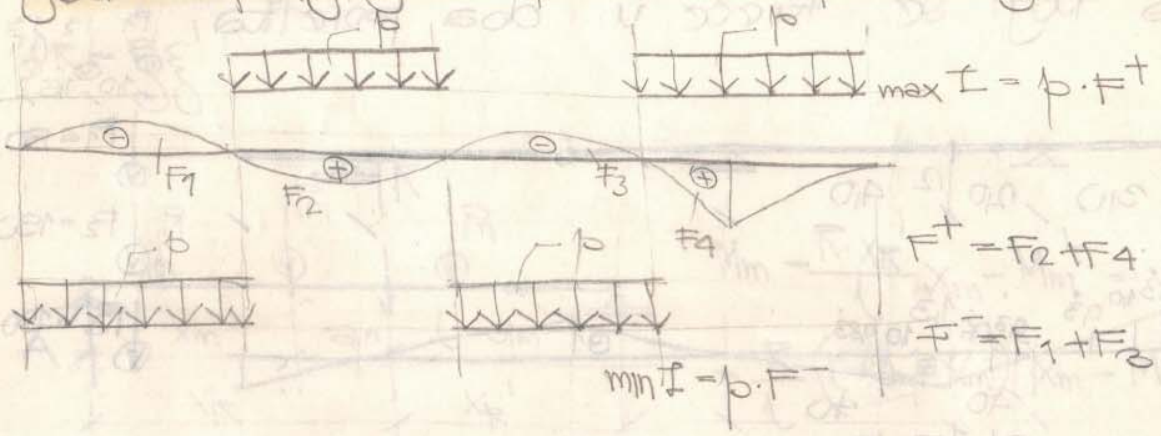
Модул: Конструкције

– материјал за вежбе –

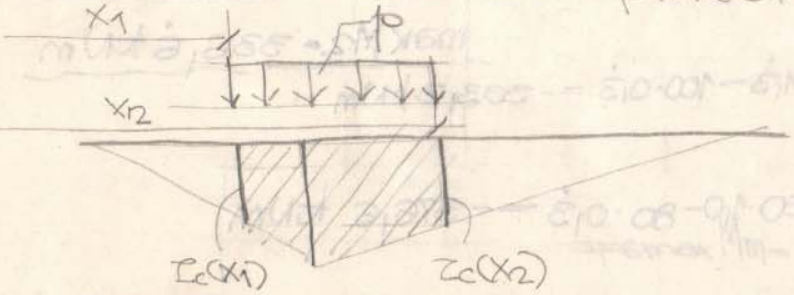
2024.

MJERODAVAN POLOŽAJ, I EKSTREMNE VRIJEDNOSTI

a) jednoliko podijeljeno pokretno opterećenje



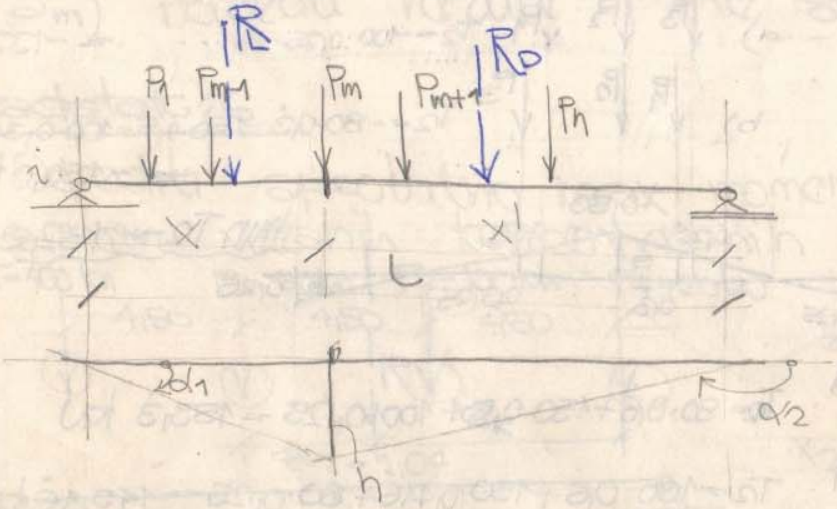
= konačna dužina opterećenja



uslov:

$$T_c(x_1) = T_c(x_2)$$

b) pokretan sistem vertikalnih koncentrisanih sila



uslov:

$$\sum_{m=1}^{n} P_m \operatorname{tg}\alpha_m = 0$$

$$R_L \cdot \operatorname{tg}\alpha_1 + R_D \operatorname{tg}\alpha_2 = 0 \qquad \operatorname{tg}\alpha_1 = \frac{h}{x}$$

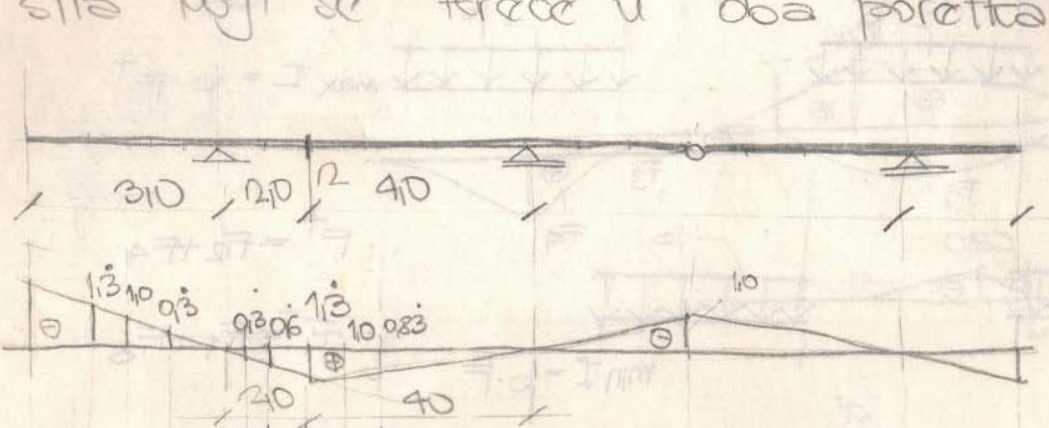
$$R_L \frac{h}{x} = R_D \frac{h}{x'} = \frac{R_L + R_D}{x + x'} = \qquad \operatorname{tg}\alpha_2 = -\frac{h}{x'}$$

$$= \frac{R}{L}$$

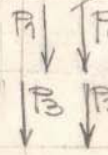
pom. udesno → $\frac{R_L}{x} < \frac{R}{L} < \frac{R_D}{x'}$ ← pom ulijevo $\frac{R_L}{x} > \frac{R}{L} > \frac{R_D}{x'}$

zadatak 5

Odrediti ekstremne vred. momenta i transfer-zalne sile u (pres 2) za sistem koncentrisanih sila koji se kreće u oba pravca



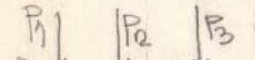
za max { a) b)



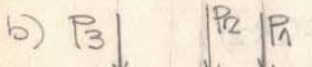
$M_2 = 80 \cdot 0{,}6 + 150 \cdot 1{,}3\dot{} + 100 \cdot 0{,}8\dot{3} = 336{,}\dot{6}$ kNm

$M_2 = 100 \cdot 0{,}3\dot{} + 150 \cdot 1{,}3\dot{} + 80 \cdot 1{,}10 = 313{,}\dot{3}$ kNm

$\max M_2 = 336{,}\dot{6}$ kNm

a) 

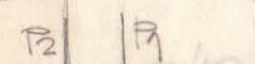
$M_2 = -80 \cdot 2 - 150 \cdot 1{,}3\dot{} - 100 \cdot 0{,}3\dot{} = -393{,}\dot{3}$ kNm

b) 

$M_2 = -100 \cdot 2 - 150 \cdot 1{,}10 - 80 \cdot 0{,}3\dot{} = -376{,}6$ kNm

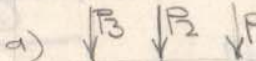
c) 

$M_2 = -150 \cdot 2 - 100 \cdot 1{,}10 = -400$ kNm

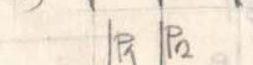


$M_2 = -150 \cdot 2 - 80 \cdot 1{,}3\dot{} = -406{,}\dot{6}$ kNm

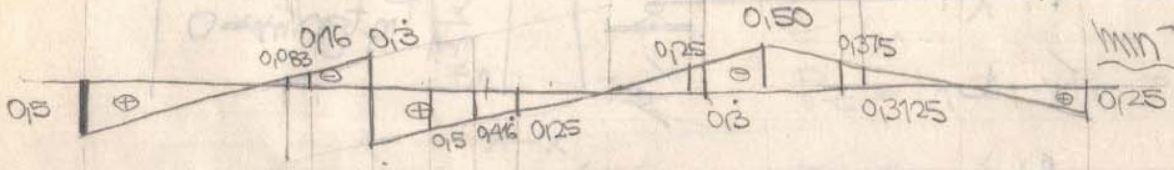
$\min M_2 = -406{,}\dot{6}$ kNm

a) 

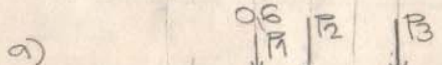
$T_2 = -100 \cdot 0{,}25 - \ldots = -132{,}9$ kN

b) 

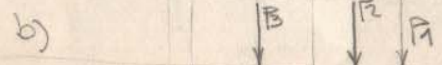
$T_2 = -80 \cdot 0{,}\dot{6} - 150 \cdot 0{,}5 - 100 \cdot 0{,}3125 = -130{,}0$ kN



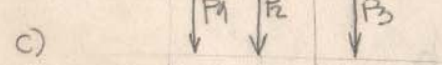
$\min T_2 = -132{,}9$ kN

a) 

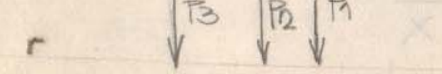
$T_2 = 80 \cdot 0{,}\dot{6} + 150 \cdot 0{,}5 + 100 \cdot 0{,}25 = 153{,}\dot{3}$ kN

b) 

$T_2 = 100 \cdot 0{,}\dot{6} + 150 \cdot 0{,}41\dot{6} + 80 \cdot 0{,}25 = 149{,}1\dot{6}$ kN

c) 

$T_2 = -80 \cdot 0{,}16\dot{} + 150 \cdot 0{,}\dot{6} + 100 \cdot 0{,}41\dot{6} = 128{,}\dot{3}$ kN

r 

$T_2 = -100 \cdot 0{,}08\dot{3} + 150 \cdot 0{,}\dot{6} + 80 \cdot 0{,}5 = 131{,}\dot{6}$ kN

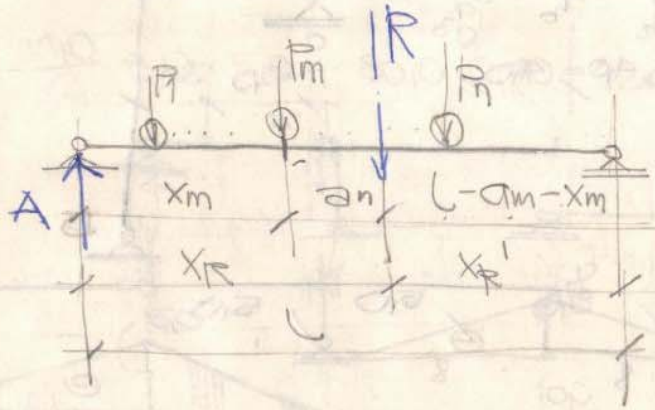
$\max = T_2 = 153{,}\dot{3}$ kN

GRANIČNA VRIJEDNOST MOMENTA SAVIJANJA

a) $\text{aps}_{max} M = \frac{p \cdot l^2}{2} \omega_R$ — ravnomerno pod. opterećenje

$\omega_R = \int_0^{\xi} \xi'$

b)



$M_m = \frac{R \cdot x_R^1}{l} x_m - M_m^L =$

$= \frac{R}{l}(l - a_m - x_m) \cdot x_m - M_m^L =$

$\frac{dM_m}{dx} = \frac{R}{l}(l - a_m - 2x_m) = 0$

$x_m = \frac{l - a_m}{2}$

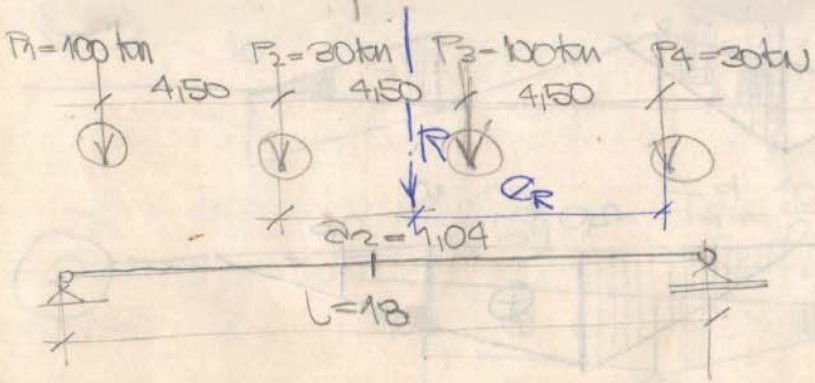
$x_R' = \frac{l - a_m}{2}$ - sa stuce

$\text{aps}_{max} M_m = \frac{R}{l}\left(\frac{l - a_m}{2}\right)^2 - M_m^L$

aps. max moment savijanja ispod sila Pm javlja se tada sredina grede polovi odstojanje (am) između rezult. i te sile.

zadatak:6

Odrediti apsolutno max moment prost. grede



$e_R = \frac{200 \cdot 9,0 + 60 \cdot 4,5}{260} = 7,38m$

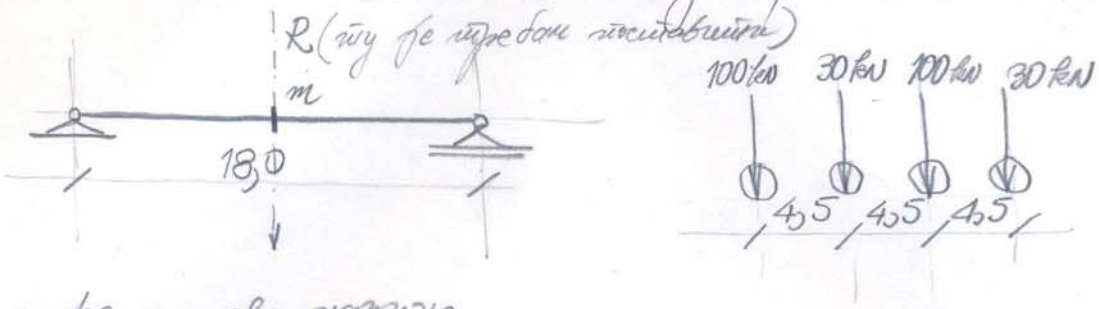
$x_2 = \frac{18 - 1,04}{2} = 8,48$

$\text{aps}_{max} M = \frac{260}{18} 8,48^2 - 100 \cdot 4,5 =$

$\text{aps}_{max} M = 588,706 \text{ kNm}$

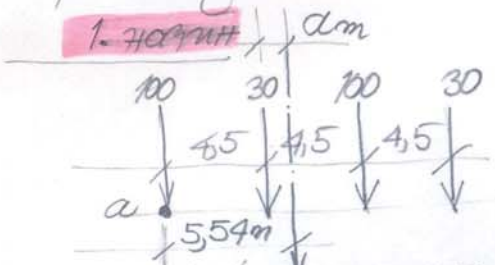
ОБРАТИ ПАЖЊУ

Одредити апсолутни максимални моменат греде за систем вертикалних концентрисаних сила



– можемо на два начина

1. начин



$R = 100+30+100+30 = 260,0\ kN$

– положај резултанте тражимо

$\Sigma M_a = 30\cdot 4,5 + 100\cdot 9,0 + 30\cdot 13,5 = 1440,0\ kNm$

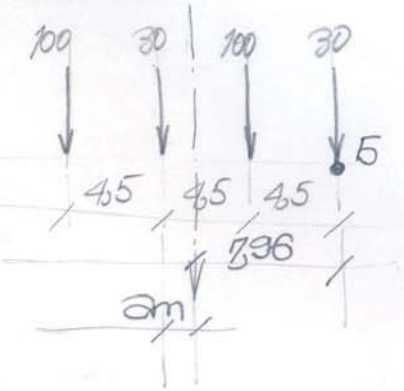
$x_R = \frac{\Sigma M_a}{R} = \frac{1440}{260} = 5,54\ m$

$a_m = 5,54 - 4,50 = 1,04\ m$

$* M_m^l = 100\cdot 4,5 = 450,0\ kNm$ (тражи се моменат за тачку силе најближој резултанти.)

$$\text{апс. max } M_m = \frac{R}{l}\left(\frac{l-a_m}{2}\right)^2 - M_m^l = \frac{260}{18,0}\left(\frac{18,0-1,04}{2}\right)^2 - 450,0 = 588,71\ kNm$$

2. начин



$R = 100+30+100+30 = 260,0\ kN$

положај резултанте

$\Sigma M_B = 100\cdot 4,5 + 30\cdot 9,0 + 100\cdot 13,5 = 2070,0\ kNm$

$x_R = \frac{\Sigma M_B}{R} = \frac{2070}{260} = 7,96\ m$

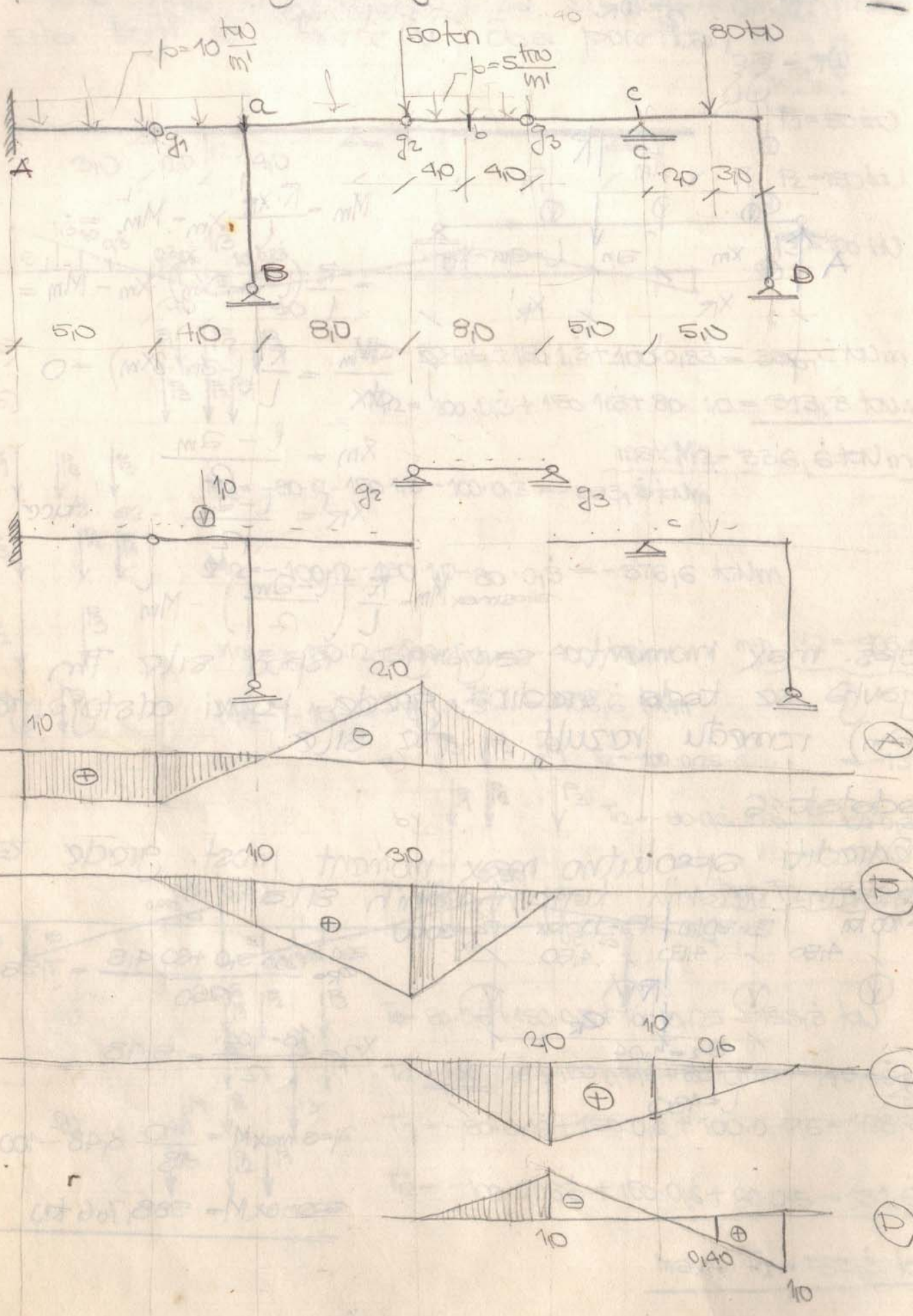
$a_m = 9,00 - 7,96 - 1,04\ m$

$M_m^l = 100\cdot 4,5 = 450,0\ kNm$

$$\text{апс. max } M_m = \frac{260}{18,0}\left(\frac{18,0-1,04}{2}\right)^2 - 450,0 = 588,71\ kNm$$

$g = \frac{x}{2}$

Proračunati reakcije i sile u presjecima a, b, c pomoću utjecajnih linija.

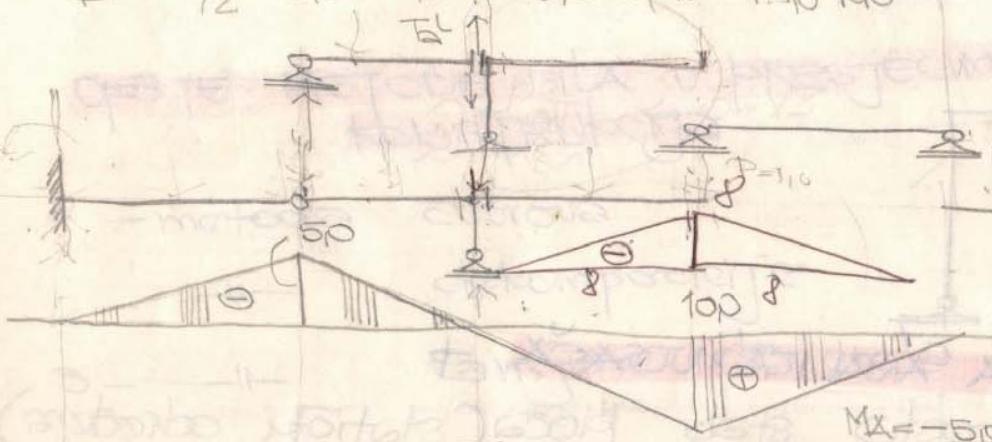


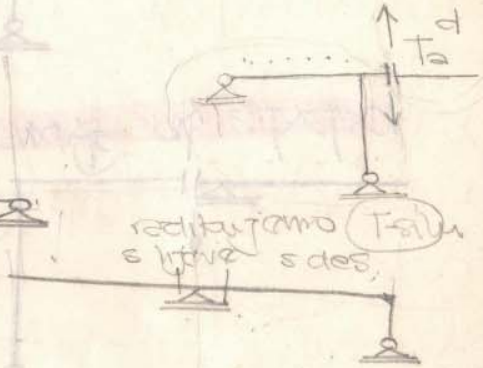
$A = \left(1{,}10 \cdot 5{,}0 + \frac{4{,}10}{2} \cdot 1{,}10\right) \cdot 10 - \frac{2{,}0}{2} \cdot 8{,}0 \cdot 5{,}10 - 5{,}0 \cdot 2{,}0 = -70\ \text{tN}$

$B = \frac{1{,}10 \cdot 4{,}10}{2} \cdot 10{,}0 + \frac{3{,}10}{2} \cdot 8{,}0 \cdot 5{,}0 + 50 \cdot 3{,}10 = 230\ \text{tN}$

$C = \frac{2{,}0 \cdot 8{,}0}{2} \cdot 5{,}10 + 80{,}0 \cdot 0{,}60 = 88{,}0\ \text{tN}$

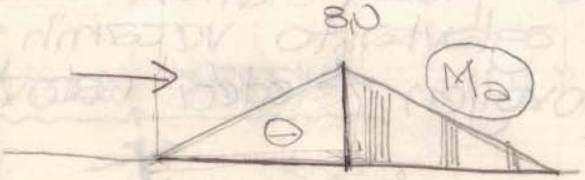
$D = -\frac{1{,}0}{2}\ 8{,}0 \cdot 5{,}10 + 80{,}0 \cdot 0{,}40 = 12{,}0\ \text{tN}$





(M_A)

$M_A = -\frac{5{,}0}{2} \cdot 5{,}0 \cdot 10{,}0 + \frac{10{,}0}{2} \cdot 8{,}0 \cdot 5{,}0 + +50{,}0 \cdot 10{,}0 = 475\ \text{tNm}$



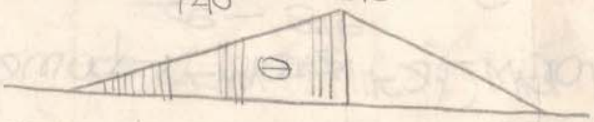
$M_B = -\frac{8{,}0}{2}\ 80\ 5{,}0 - 50 \cdot 8{,}0 = -560\ \text{tNm}$

$\frac{8{,}4}{2} \cdot 10 = 180$

$M_b = \frac{2{,}10}{2} \cdot 80 \cdot 5{,}10 = 90\ \text{tNm}$

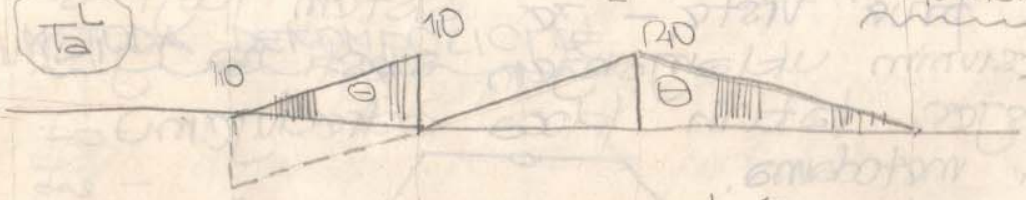
(M_b)

M_c



$M_C = -\frac{5{,}10 \cdot 8{,}0}{2} \cdot 5{,}10 = -100{,}0\ \text{tNm}$

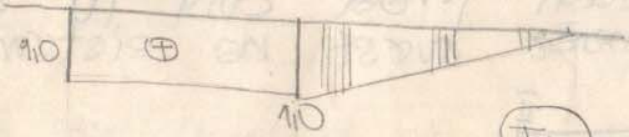
T_a^L



$T_a^L = -\frac{1{,}10}{2}\ 40 \cdot 10{,}0 - \frac{2{,}10}{2} \cdot 80\ 50 - 50 \cdot 2{,}10 = -160{,}0\ \text{tN}$

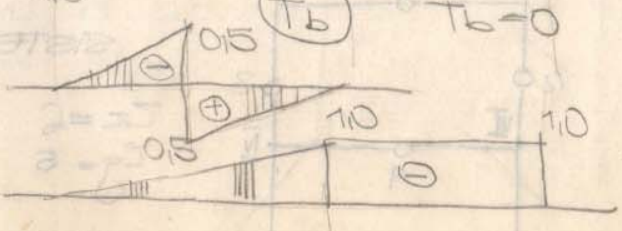
T_a^d

$T_a^d = \frac{1{,}0}{2} \cdot 80 \cdot 5{,}10 + 50{,}0 \cdot 1{,}0 = 70\ \text{tN}$



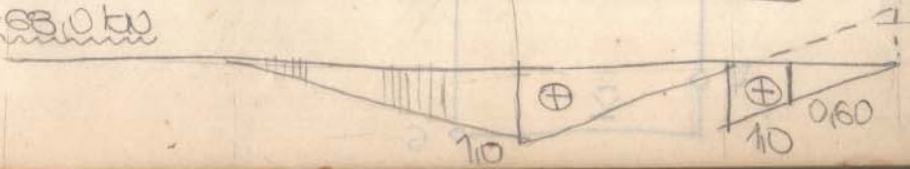
T_b

$T_b = 0$



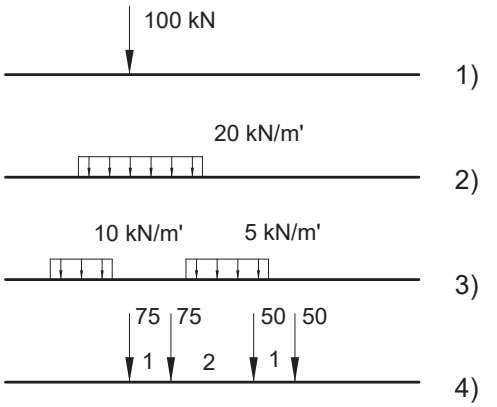
$T_c^L = -\frac{1{,}0}{2} \cdot 8{,}0 \cdot 5{,}0 = -20\ \text{tN}$

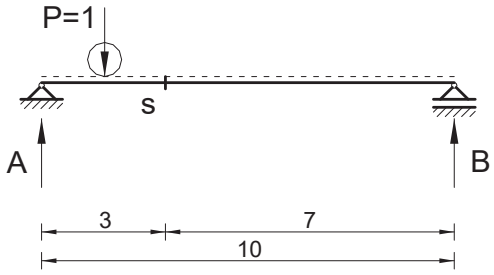
$T_c^d = \frac{1{,}0}{2}\ 80 \cdot 5{,}0 + 80 \cdot 0{,}60 = 68{,}0\ \text{tN}$



Пример

За носач приказан на скици са оптерећењима (1,2,3,4) одредити екстремне вредности момента савијања у пресеку (s).

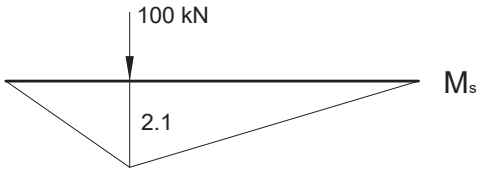




1.|

$maxM_s = P \cdot Z(s,u) = 100 \cdot 2.1 = 210.00kNm$

$minM_s = 0$



2.|

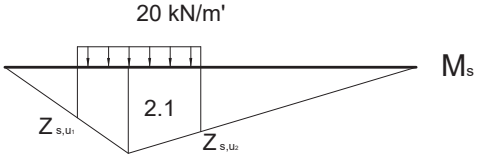
Услов из кога следи, $maxM_s$ је:

$Z(s,u_1) = Z(s,u_2)$, односно: $minM_s = 0$

На основу Талесове теореме (сличности троуглова), имамо:

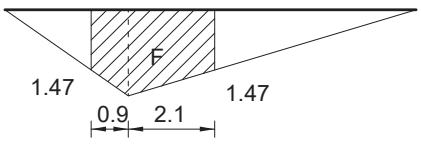
$$\frac{Z(s,u_1)}{x} = \frac{\frac{x \cdot x'}{l}}{x_s} = \frac{2.1}{3} \cdot x, \text{ односно}$$

$$\frac{Z(s,u_2)}{7-x} = \frac{\frac{x \cdot x'}{l}}{x'_s} = \frac{2.1}{7} \cdot (7-x)$$



$$\frac{2.1}{3} \cdot x = \frac{2.1}{7} \cdot (7-x)$$

$$x = 2.1m$$



$$Z(s,u_1) = \frac{2.1}{3} \cdot 2.1 = 1.47$$

$$Z(s,u_2) = \frac{2.1}{7} \cdot (7-2.1) = 1.47$$

$$maxM_s = p \cdot F =$$

$$= 20 \cdot \left[\left(\frac{1.47+2.1}{2} \right) \cdot 0.9 + \left(\frac{1.47+2.1}{2} \right) \cdot 2.1 \right] =$$

$$= 20 \cdot (1.607 + 3.749) =$$

$$= 20 \cdot 5.355 = 107.10kNm$$

3.|

Положај 1: $0 \leq x_1 \leq 1.5$ (p_1- лево, p_2 - десно)

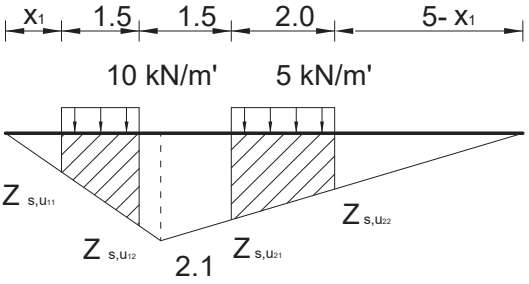
$$\sum p_m \cdot Z_{(s,u_{m1})} = \sum p_m \cdot Z_{(s,u_{m2})}$$

$$p_1: \quad Z_{(s,u_{11})} = \frac{2.1}{3} \cdot x_1$$

$$Z_{(s,u_{12})} = \frac{2.1}{3} \cdot (x_1 + 1.5)$$

$$p_2: \quad Z_{(s,u_{21})} = \frac{2.1}{7} \cdot (7 - x_1)$$

$$Z_{(s,u_{22})} = \frac{2.1}{7} \cdot (5 - x_1)$$



$$10 \cdot \frac{2.1}{3} \cdot x_1 + 5 \cdot \frac{2.1}{7} \cdot (7 - x_1) = 10 \cdot \frac{2.1}{3} \cdot (x_1 + 1.5) + 5 \cdot \frac{2.1}{7} \cdot (5 - x_1)$$

$$7 \cdot x_1 + 10.5 - 1.5 \cdot x_1 = 7 \cdot x_1 + 10.5 + 7.5 - 1.5 \cdot x_1$$

$$0 \neq 7.5$$

Положај 2: $1.5 \leq x_2 \leq 3$ (p_1- лево и десно, p_2 - десно)

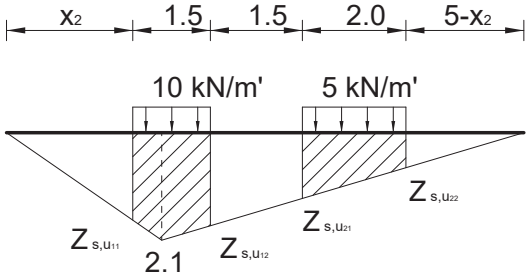
$$\sum p_m \cdot Z_{(s,u_{m1})} = \sum p_m \cdot Z_{(s,u_{m2})}$$

$$p_1: \quad Z_{(s,u_{11})} = \frac{2.1}{3} \cdot x_2$$

$$Z_{(s,u_{12})} = \frac{2.1}{7} \cdot (8.5 - x_2)$$

$$p_2: \quad Z_{(s,u_{21})} = \frac{2.1}{7} \cdot (7 - x_2)$$

$$Z_{(s,u_{22})} = \frac{2.1}{7} \cdot (5 - x_2)$$



$$10 \cdot \frac{2.1}{3} \cdot x_2 + 5 \cdot \frac{2.1}{7} \cdot (7 - x_2) = 10 \cdot \frac{2.1}{7} \cdot (8.5 - x_2) + 5 \cdot \frac{2.1}{7} \cdot (5 - x_2)$$

$$7 \cdot x_2 + 10.5 - 1.5 \cdot x_2 = 25.5 - 3 \cdot x_2 + 7.5 - 1.5 \cdot x_2$$

$$10 \cdot x_2 = 22.5$$

$$x_2 = 2.25$$

$$maxM_s = \sum p_m \cdot F_m =$$

$$= 10 \cdot \left(\frac{1.575 + 2.1}{2} + \frac{1.875 + 2.1}{2} \right) \cdot 0.75 + 5 \cdot \left(\frac{1.425 + 0.825}{2} \right) \cdot 2 =$$

$$= 28.69 + 11.25 = 39.94 kNm$$

$$minM_s = 0$$

Положај 3: $3 \leq x_2 \leq 10$ (p_1, p_2 – на десно)

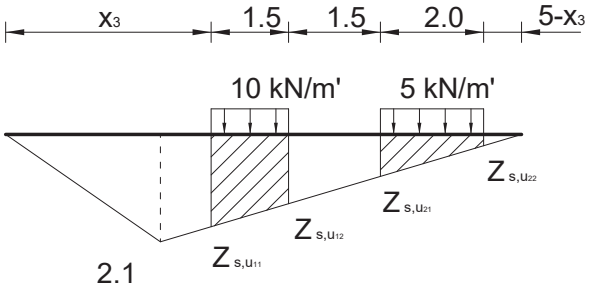
$$\sum p_m \cdot Z_{(s,u_{m1})} = \sum p_m \cdot Z_{(s,u_{m2})}$$

p_1 : $Z_{(s,u_{11})} = \frac{2.1}{7} \cdot (10 - x_3)$

$Z_{(s,u_{12})} = \frac{2.1}{7} \cdot (8.5 - x_3)$

p_2 : $Z_{(s,u_{21})} = \frac{2.1}{7} \cdot (7 - x_3)$

$Z_{(s,u_{22})} = \frac{2.1}{7} \cdot (5 - x_3)$



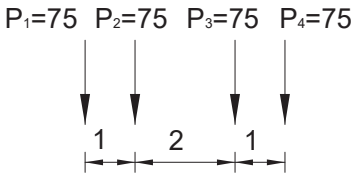
$$10 \cdot \frac{2.1}{3} \cdot x_3 + 5 \cdot \frac{2.1}{7} \cdot (7 - x_3) = 10 \cdot \frac{2.1}{7} \cdot (8.5 - x_3) + 5 \cdot \frac{2.1}{7} \cdot (5 - x_3)$$

$$40.5 \neq 33$$

4.|

Услов је: $\frac{R}{l} > \frac{R^L}{x_s}$

$> \frac{R^D}{x_s'}$

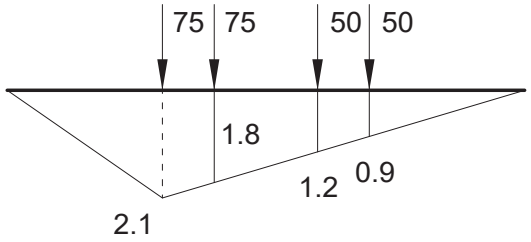


$R = \sum P_m =$

$= 75 + 75 + 50 + 50 = 250kN$

$\frac{R}{l} = \frac{250}{10} = 25\frac{kN}{m'}$

Положај 1: меродавна сила је P_1



$\frac{R^L}{x_s} = \frac{0}{3} = 0 < \frac{R}{l}$

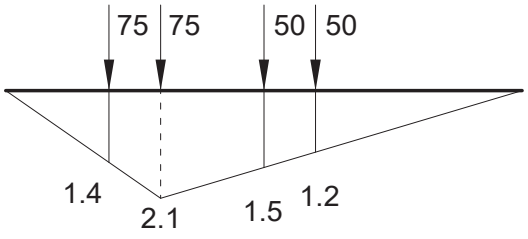
$\frac{R^D}{x_s'} = \frac{75 + 50 + 50}{7} = 25\frac{kN}{m'} = \frac{R}{l}$

$minM_s = 0$

$maxM_s = 2.1 \cdot 75 + 1.8 \cdot 75 + 1.2 \cdot 50 + 0.9 \cdot 50 =$

$= 397.50kNm$

Положај 2: меродавна сила је P_2



$\frac{R^L}{x_s} = \frac{75}{3} = 25\frac{kN}{m'} = \frac{R}{l}$

$\frac{R^D}{x_s'} = \frac{50 + 50}{7} = 14.29\frac{kN}{m'} < \frac{R}{l}$

$minM_s = 0$

$maxM_s = 1.4 \cdot 75 + 2.1 \cdot 75 + 1.2 \cdot 50 =$

$= 397.50kNm$

Положај 3: меродавна сила је P_3

$\frac{R^L}{x_s} = \frac{125}{3} = 41.66\frac{kN}{m'} > 25 = \frac{R}{l}$ **не може** бити меродаван положај оптерећења.!.

Међуположај 1-2: (једна проба)

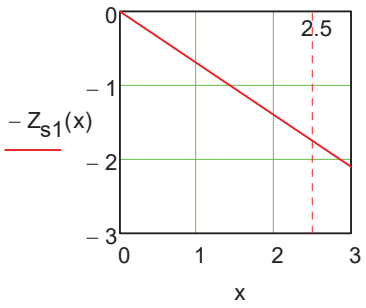
$minM_s = 0$

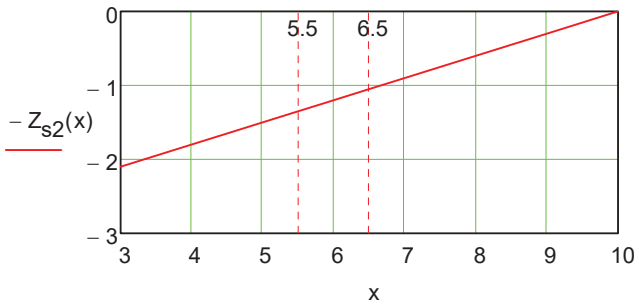
$maxM_s = 1.75 \cdot 75 + 1.95 \cdot 75 + 1.35 \cdot 50 + 1.05 \cdot 50 =$

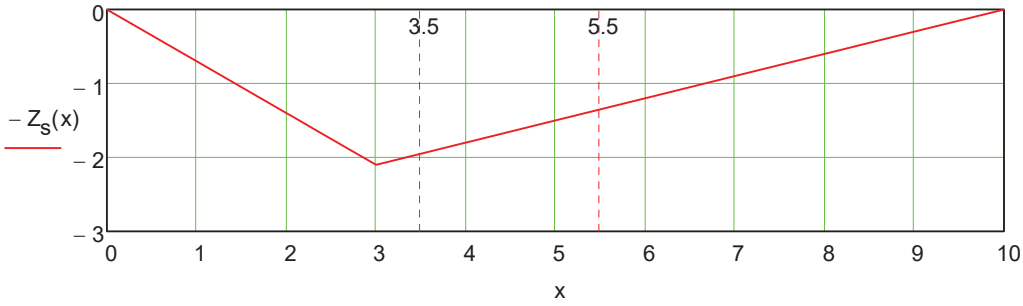
$= 397.50kNm$

Функције утицајне линије лево и десно у односу на посматрани пресек "s"

$Z_{s1}(x) := 0.7 \cdot x$ $\quad Z_{s2}(x) := (-0.3 \cdot x + 2.1) + 0.9$ $\quad Z_s(x) := \begin{cases} Z_{s1}(x) & \text{if } 0 \le 3 \\ Z_{s2}(x) & \text{if } 3 \le x \le 10 \end{cases}$





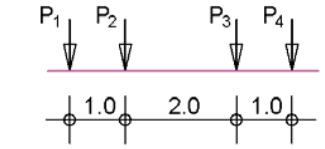


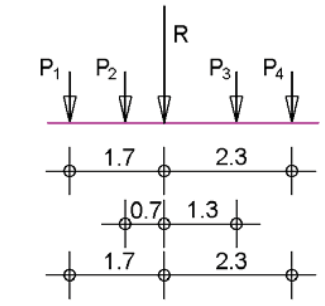
$Z_s(0) = 0$ $\quad Z_s(2.5) = 1.75$ $\quad Z_s(3) = 2.1$ $\quad Z_s(3.5) = 1.95$

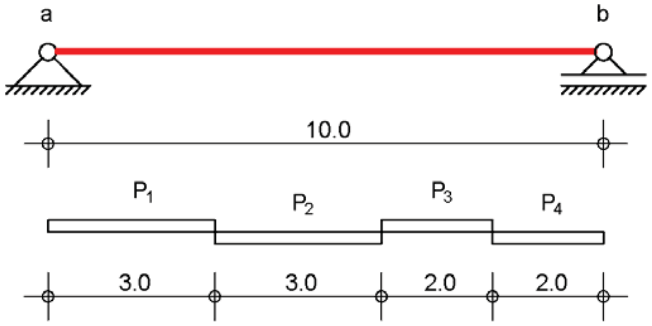
$Z_s(5.5) = 1.35$ $\quad Z_s(6.5) = 1.05$ $\quad Z_s(10.0) = 0$

Пример

За носач приказан на скици услед датог покретног система везаних концентрисаних сила одредити дијаграм екстремних вредности момената савијања.







$P_1 := 75 \qquad P_2 := 75$

$P_3 := 50 \qquad P_4 := 50$

$R := P_1 + P_2 + P_3 + P_4$

$R = 250$

$$e_R := \frac{P_2 \cdot 1 + P_3 \cdot 3 + P_4 \cdot 4}{R}$$

$e_R = 1.7$

Dužine segmenata na kojima su određene sile merodavne:

$$\lambda_m = \frac{P_m}{R} L$$

$$\lambda_1 := \frac{75}{250} \cdot 10 = 3$$

$$\lambda_2 := \frac{75}{250} \cdot 10 = 3$$

$$\lambda_3 := \frac{50}{250} \cdot 10 = 2$$

$$\lambda_4 := \frac{50}{250} \cdot 10 = 2$$

Segment I (merodavna je sila P1, 0<x1<3)

Prva nulta tačka parabole sa leve strane u odnosu na maxMm

$$\frac{L - a_1}{2} = \frac{10 - 1.7}{2} = 4.15$$

Druga nulta tačka parabole sa desne strane u odnosu na maxMm

$$\sqrt{\left(\frac{L - a_1}{2}\right)^2 - \frac{P_1 \cdot e_1 \cdot L}{R}} = \sqrt{\left(\frac{10 - 1.7}{2}\right)^2 - \frac{0 \cdot 1.7 \cdot 10}{250}} = 4.15$$

$Mm\_levo := 0 \qquad a\_m := 1.7 \qquad L := 10 \qquad R := 250 \qquad x := 0, 0.1 .. 10$

$$maxM\_m := \frac{R}{L}\cdot\left(\frac{L - a\_m}{2}\right)^2 - Mm\_levo \qquad maxM\_m = 430.563$$

$$A(x1) := \frac{(10 - 1.7) - x1}{10}\cdot 250 \qquad M1(x1) := A(x1)\cdot x1 \qquad M1(x1) \rightarrow -x1\cdot(25\cdot x1 - 207.5)$$

$$\frac{d}{dx1}M1(x1) \rightarrow 20\,x1 := \frac{207.5}{50} \qquad x1 = 4.15$$

$M1(0) = 0$

$M1(3) = 397.5$

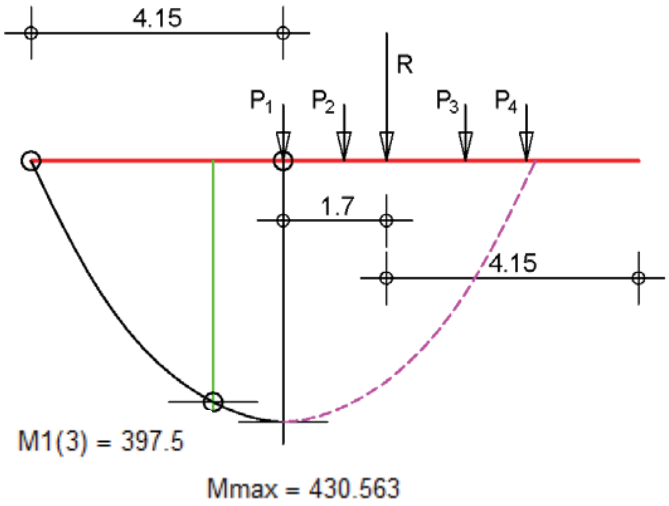
$M1(x1) = 430.563$

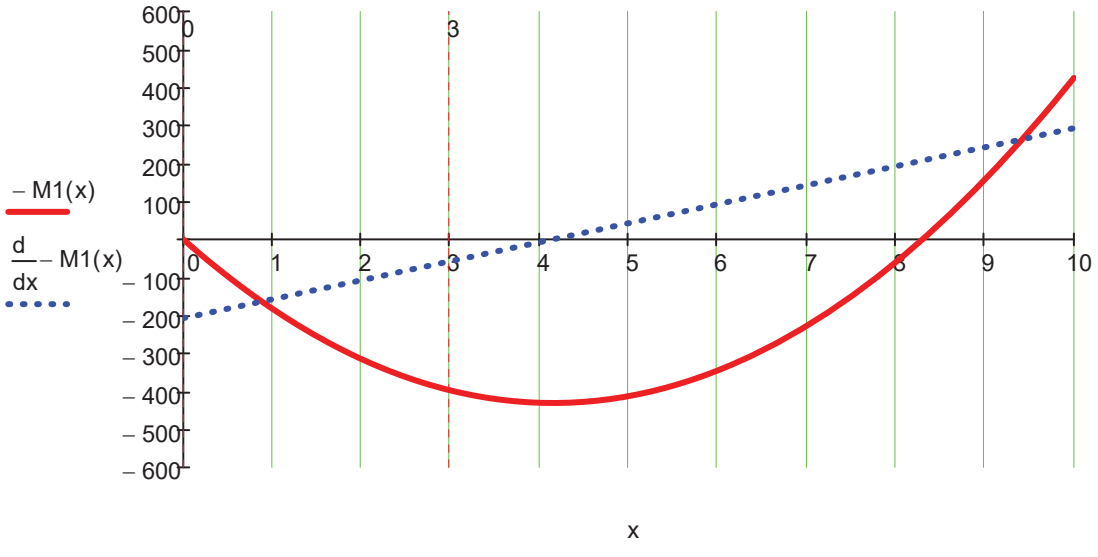
$$x_{m1} := 4.15 + \sqrt{4.15^2 - \frac{0}{250}}$$

$x_{m1} = 8.3$

$$Mmax := \frac{250}{10}\cdot\left(\frac{10 - 1.7}{2}\right)^2 - 0$$

$Mmax = 430.563$





Segment II (merodavna je sila P2, 3<x2<6)

$$A(x2) := \frac{(10 - 0.7) - x2}{10} \cdot 250$$

$$M2(x2) := A(x2) \cdot x2 - 75 \cdot 1$$

$$M2(x2) \rightarrow -x2 \cdot (25 \cdot x2 - 232.5) - 75$$

$$\frac{d}{dx2} M2(x2) \rightarrow 232.5 - 50 \cdot x2$$

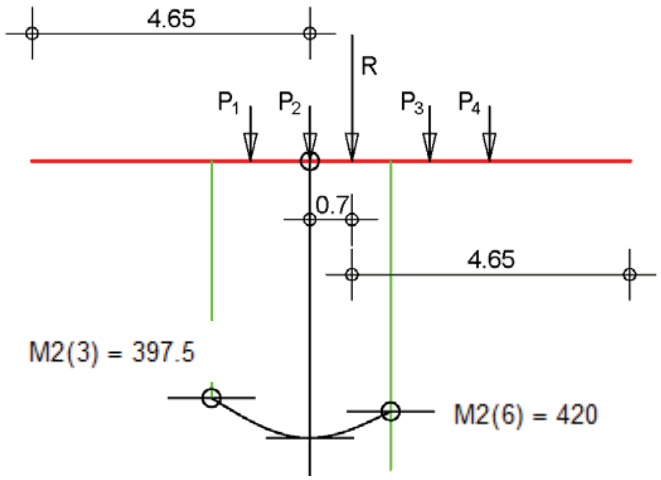
$$x2 := \frac{232.5}{50}$$

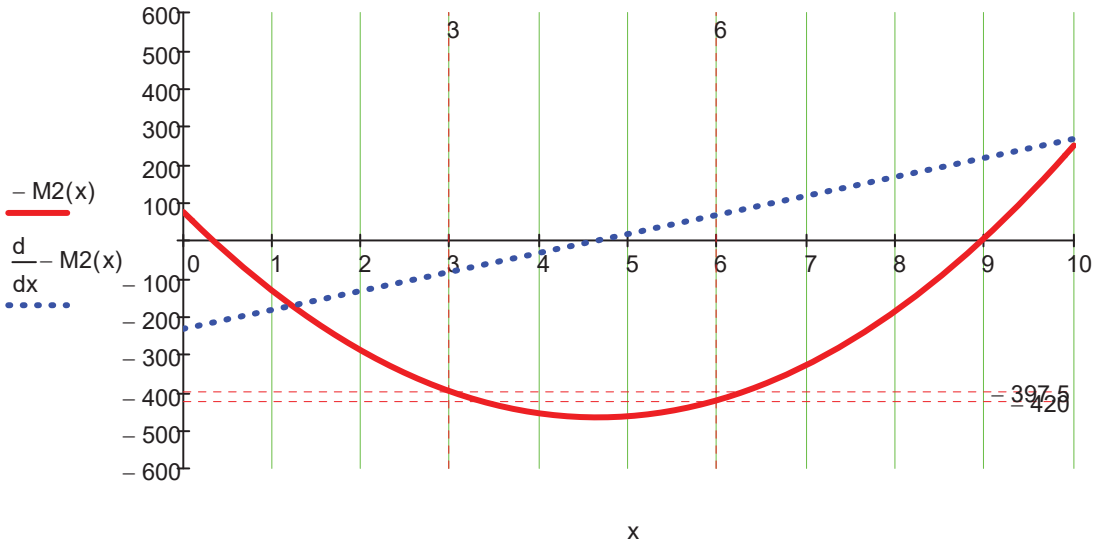
$$x2 = 4.65$$

$$M2(3) = 397.5$$

$$M2(4.65) = 465.563$$

$$M2(6) = 420$$





Segment III (merodavna je sila P3, 6<x3<8)

$$A(x3) := \frac{(10 + 1.3) - x3}{10} \cdot 250$$

$$M3(x3) := A(x3) \cdot x3 - (75 \cdot 3 + 75 \cdot 2)$$

$$\frac{d}{dx3} M3(x3) \rightarrow 282.5 - 50 \cdot x3$$

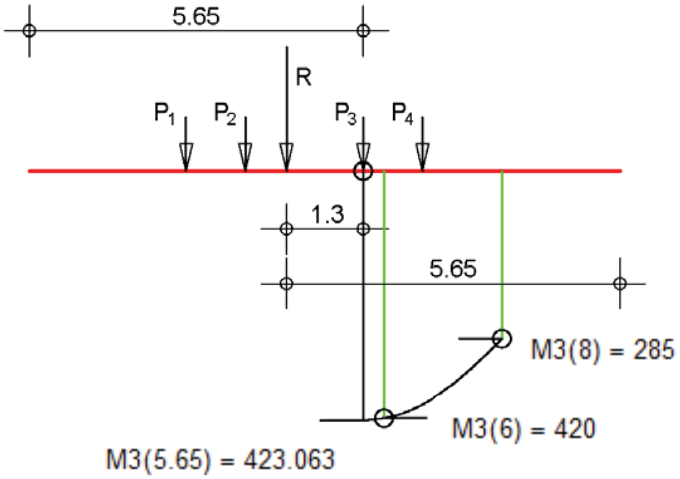
$$M3(x3) \rightarrow -x3 \cdot (25 \cdot x3 - 282.5) - 375$$

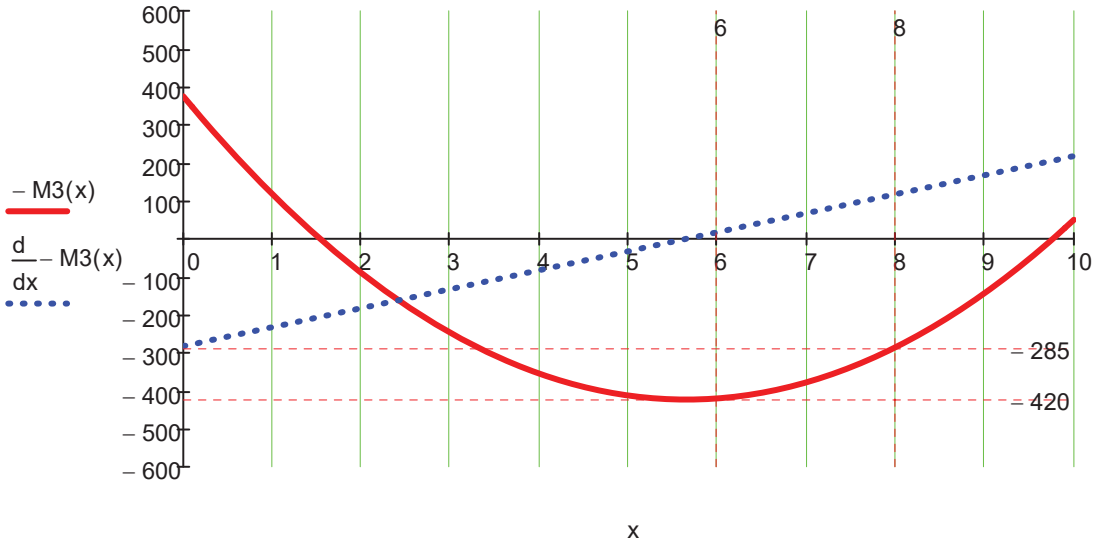
$$x3 := \frac{282.5}{50} \qquad x3 = 5.65$$

$M3(5.65) = 423.063$

$M3(6) = 420$

$M3(8) = 285$





Segment IV (merodavna je sila P4, 8<x4<10)

$$A(x4) := \frac{(10 + 2.3) - x4}{10} \cdot 250$$

$$M4(x4) := A(x4) \cdot x4 - (75 \cdot 4 + 75 \cdot 3 + 50 \cdot 1)$$

$$M4(x4) \rightarrow -x4 \cdot (25 \cdot x4 - 307.5) - 575$$

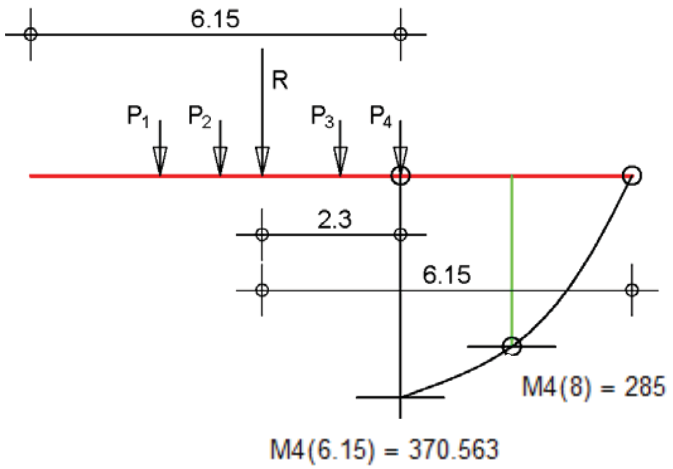
$$\frac{d}{dx4} M4(x4) \rightarrow 307.5 - 50 \cdot x4$$

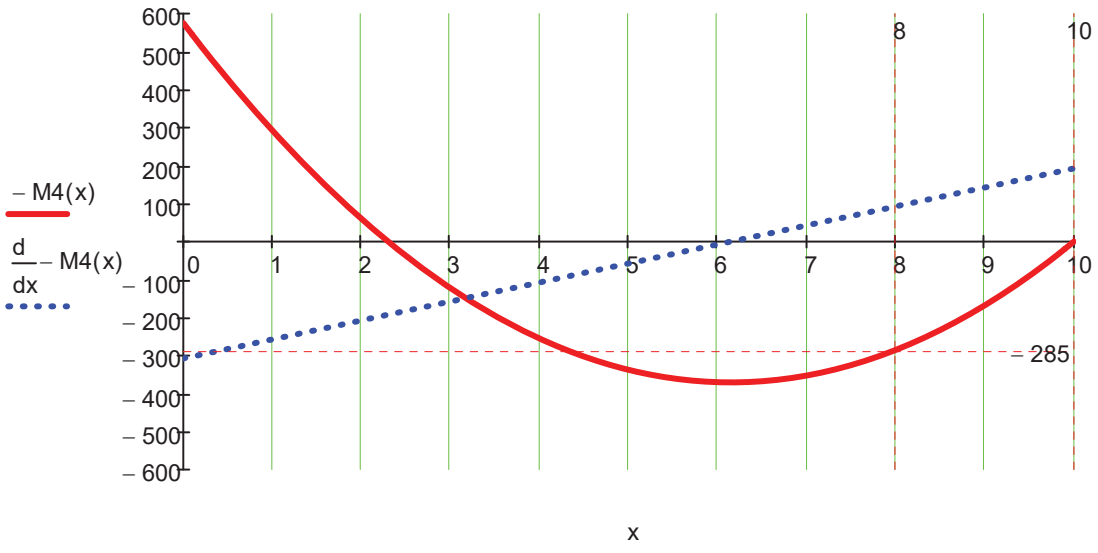
$$x4 := \frac{307.5}{50} \qquad x4 = 6.15$$

$$M4(6.15) = 370.563$$

$$M4(8) = 285$$

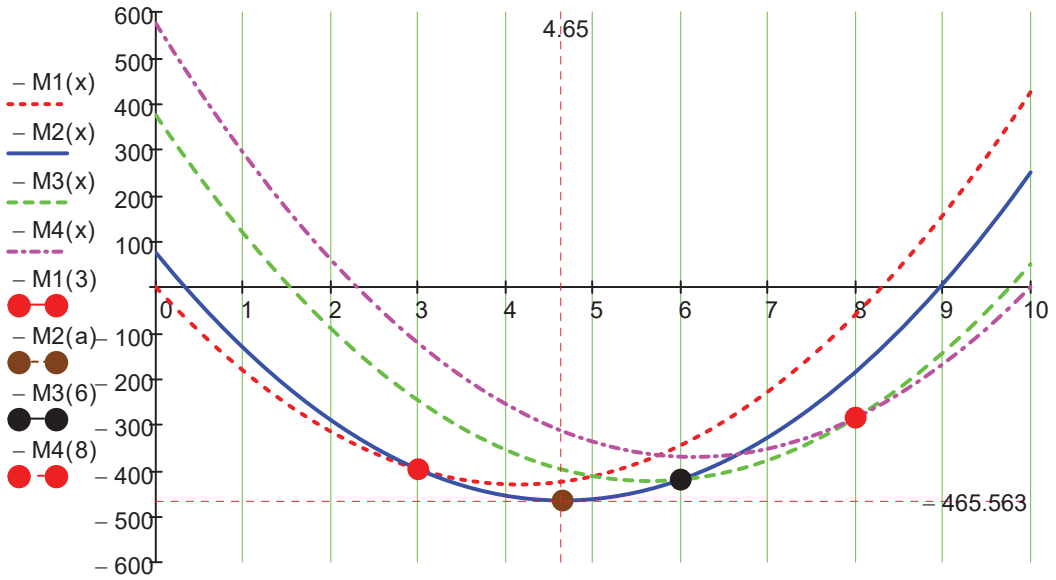
$$M4(10) = 1.137 \times 10^{-13}$$





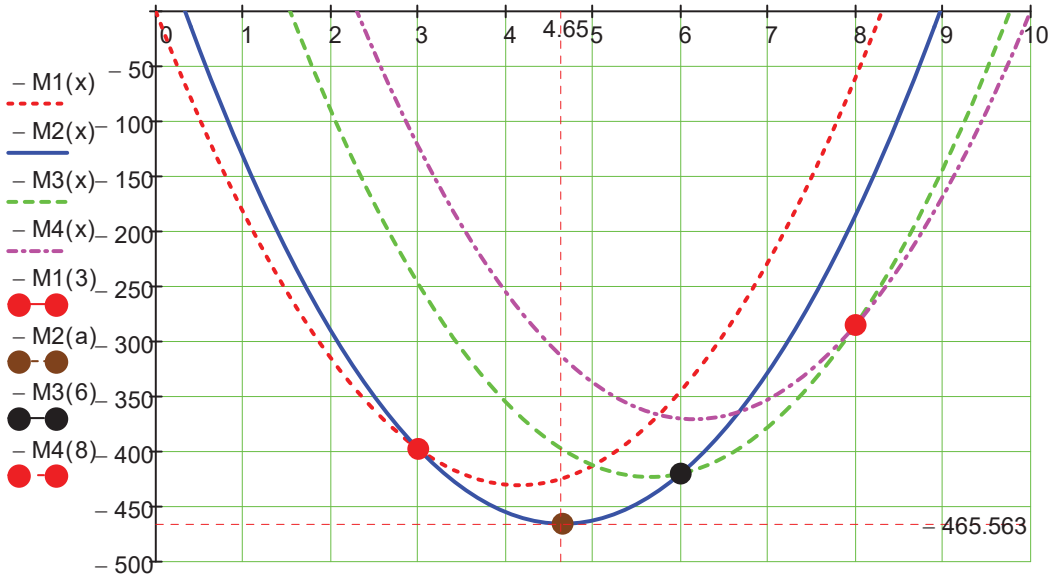
Kumulativni dijagrami ekstremnih momenata savijanja

$a := 4.65$



x, x, x, x, 3, a, 6, 8

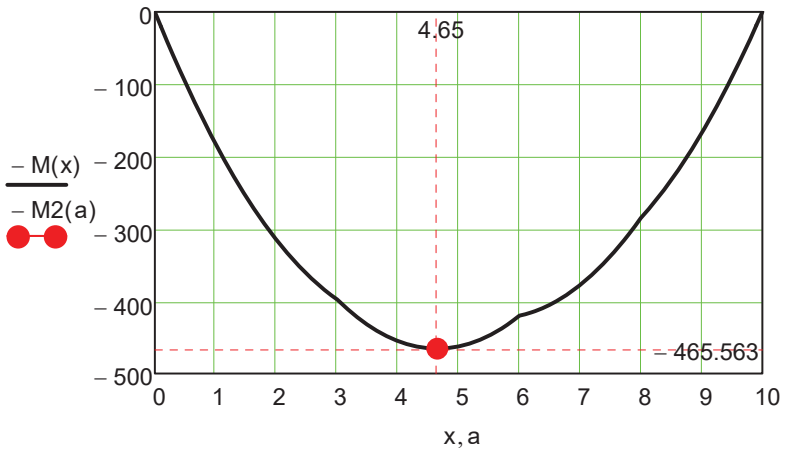
Posle odsecanja vrednost iznad "x-ose" nalazimo potrebne krive za konstrukciju anvelope.



x, x, x, x, 3, a, 6, 8

Konstruisanje anvelopa ekstremnih momenat savijanja

$$M(x) := \begin{cases} M1(x) & \text{if } 0 \le 3 \\ M2(x) & \text{if } 3 \le x \le 6 \\ M3(x) & \text{if } 6 \le x \le 8 \\ M4(x) & \text{if } 8 \le x \le 10 \end{cases}$$



M1(3) = 397.5 M2(4.65) = 465.563 M3(6) = 420 M4(8) = 285