6

СТАТИКА КОНСТРУКЦИЈА 1

Модул: Конструкције

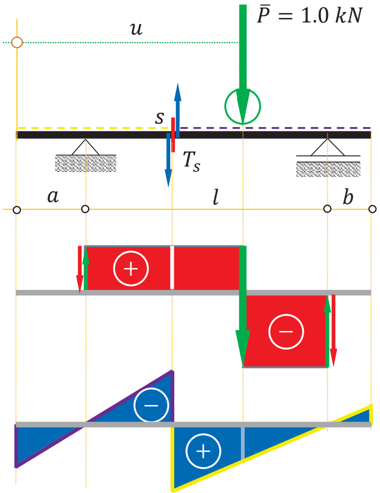
– материјал за вежбе –

2024.

Утицајне линије

Статички одређени носачи - статички утицаји

Графички приказ функције промене неког утицаја у одређеном пресеку у функцији положаја јединичне, концентрисане, покретне силе на носачу, назива се утицајна линија.



- **Непокретно** и **покретно** оптерећење
- Дијаграми утицаја и утицајне линије

$\boxed{Z(s,u)}$ утицајна функција

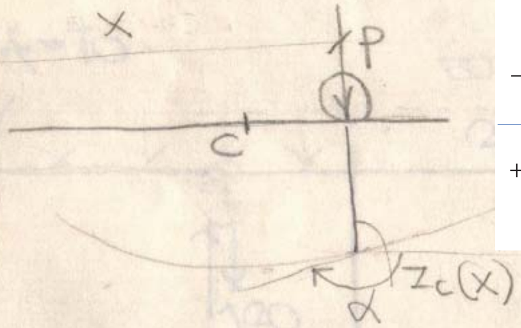
$M(s,u)$

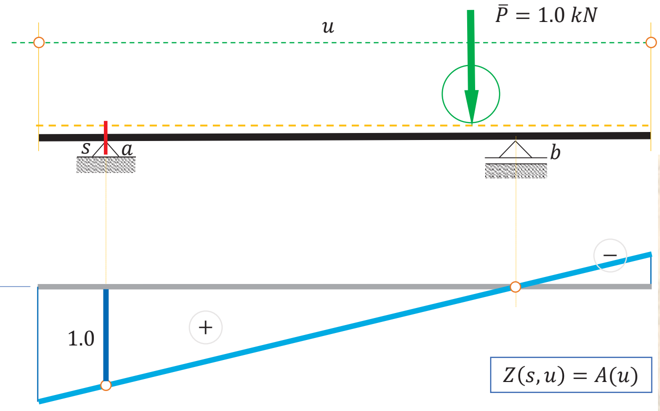
$T(s,u)$

$N(s,u)$

$\boxed{T\ (kN)}$ $\boxed{u = const. \quad Z(s,u) = Z(s)}$ дијаграм утицаја

$\boxed{T_s\ (u.l)}$ $\boxed{s = const. \quad Z(s,u) = Z(u)}$ утицајна линија





Слика 1: Конвенција - договор за "+ и –"

Срачунавање утицаја помоћу утицајне линије

$I_c = P \cdot I_c(x)$ ... za jednu koncentrisanu silu

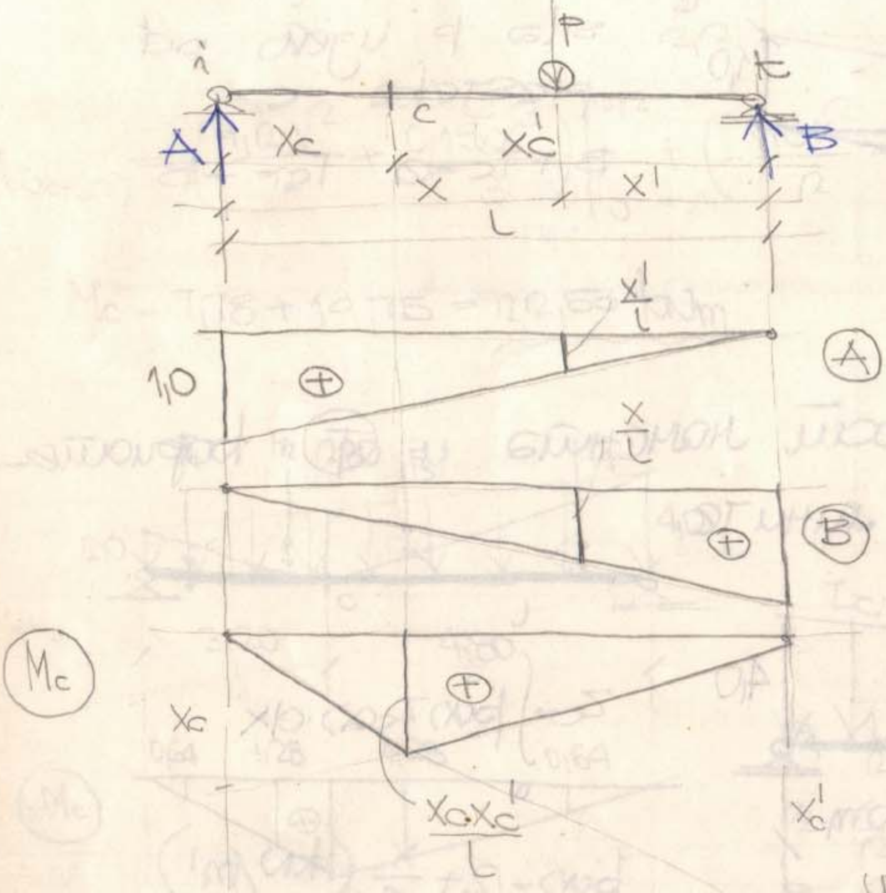
$I_c = \sum_{m=1}^{n} P_m I_c(x_m)$ ... za sistem konc. sila

$p(x) \cdot I_c(x) \cdot dx$ – elementarna sila

$I_c = \int_0^l p(x) \cdot I_c(x) \cdot dx$ – usled podijeljenog opterećenja

$T_c = M \cdot tg\alpha$

- UTICAJNE LINIJE ZA REAKCIJE I SILE U PRESJECIMA PROSTE GREDE



$\Sigma M_B = 0 \Rightarrow A = \frac{P \cdot x'}{l} = \frac{x'}{l}$

$\boxed{A = \frac{x'}{l}}$ j-na prave linije

$x' = 0 \Rightarrow A = 0$

$x' = l \Rightarrow A = 1$

$\Sigma M_A = 0 \Rightarrow B = \frac{P \cdot x}{l} = \frac{x}{l}$

$\boxed{B = \frac{x}{l}}$

$x = 0 \Rightarrow B = 0$

$x = l \Rightarrow B = 1,0$

UTICAJNA LINIJA ZA MOMENTE

sila P desno od presj

$M_c = A \cdot x_c = \frac{x' x_c}{l}$

$x' = 0 \quad M_c = 0$

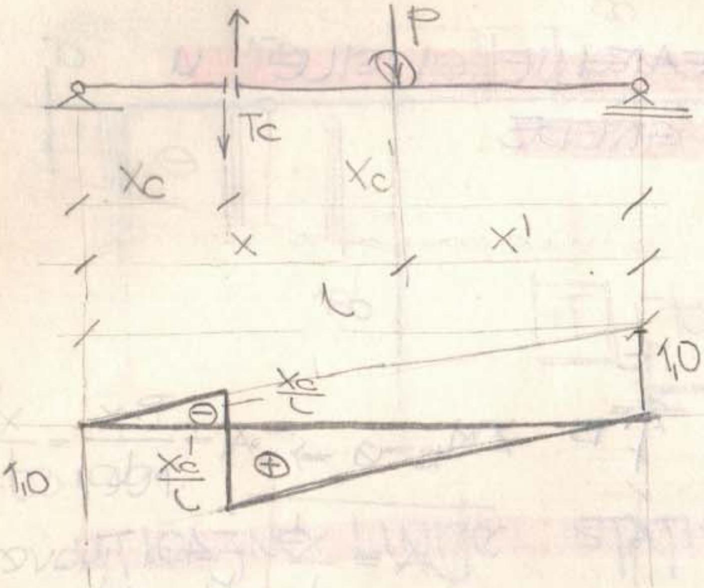
$x' = l \quad M_c = x_c$

sila P lijevo od presjeka

$M_c = B \cdot x_c' = \frac{x \cdot x_c'}{l}$

$x = 0 \Rightarrow M_c = 0$

$x = l \Rightarrow M_c = x_c'$



sila P desno od presjeka c

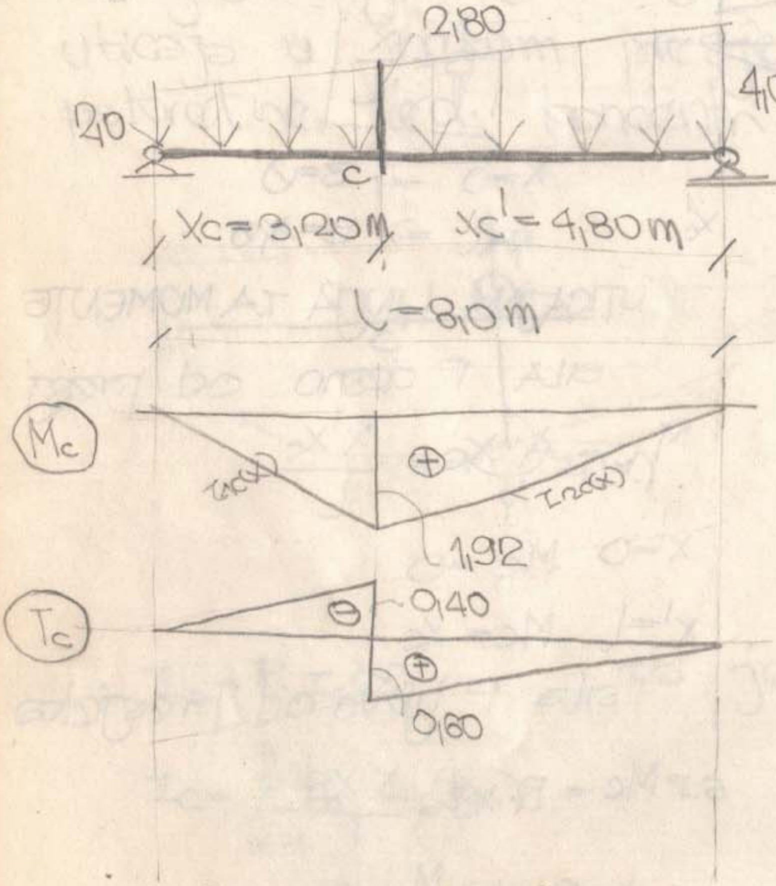
$A - T_c = 0 \Rightarrow T_c = A$

sila P lijevo od presjeka c

$B + T_c = 0 \quad T_c = -B$

задатак 3.

Одредити вриједности момената и (T) користити утицајне линије.



$$T_c = \int_0^l p(x)\, I_c(x)\, dx$$ 1.

$$p(x) = 2 + \frac{x}{4} \;(kN/m')$$

$$T_c(x_c = 3{,}20) = \int_0^{3{,}20} I_{1c}(x)\, p(x)\, dx + \int_{3{,}20}^{8} I_{2c}(x)\, p(x)\, dx$$

$$T_c(x_c = 3{,}20) = \int_0^{3{,}2} -\frac{0{,}40}{3{,}20} x \left(2 + \frac{x}{4}\right) dx +$$

$$+ \int_{3{,}2}^{8} \left(-\frac{x}{8} + 1\right)\left(2 + \frac{x}{4}\right) dx =$$

$$T_c(x_c = 3{,}2) =$$

$$\left(-0{,}25 \frac{x^2}{2} - \frac{0{,}4}{12{,}8} \frac{x^3}{3}\right)\Big|_0^{3{,}2} +$$

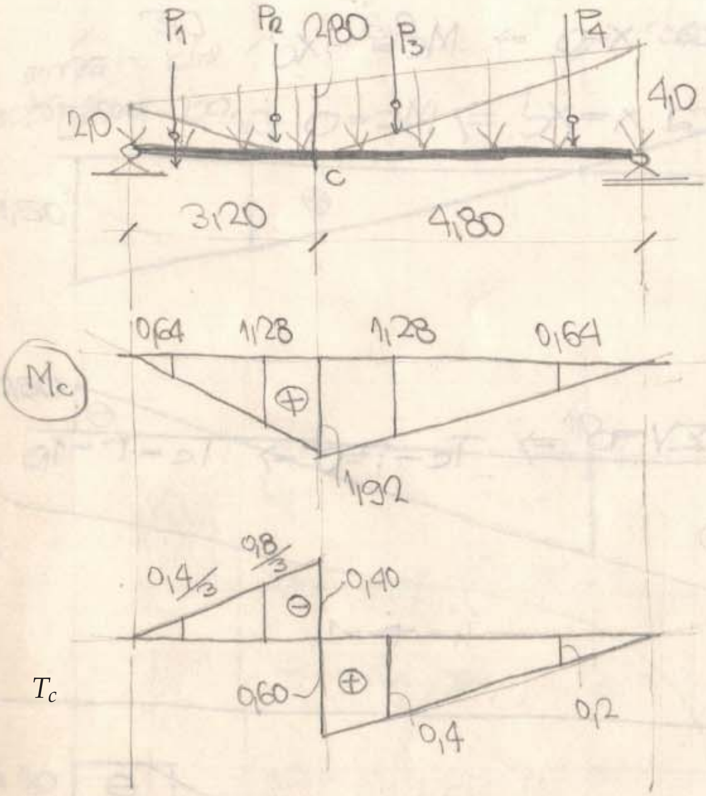
$$+\left(-0{,}125\frac{x^2}{2}-\frac{1}{32}\frac{x^3}{3}+2x+\frac{x^2}{8}\right)\Bigg|_{3,2}^{8}$$

$$T_c(x_c=3{,}20)=-1{,}62+4{,}61=\underline{2{,}99\ kN}$$

$$M_c(x_c=3{,}20)=\int_0^{3,2}\frac{1{,}92}{3{,}20}x\left(2+\frac{x}{4}\right)dx+\int_{3,2}^{8}(-0{,}4x+3{,}20)\left(2+\frac{x}{4}\right)dx=$$

$$M_c(x_c=3{,}2)=\left(\frac{1{,}2x^2}{2}+\frac{0{,}15x^3}{3}\right)\Bigg|_0^{3,2}+\left(-\frac{0{,}8x^2}{2}-\frac{0{,}1x^3}{3}+6{,}4x+\frac{9{,}8x^2}{2}\right)$$

$$M_c=7{,}78+14{,}75=\underline{22{,}53\ kNm}$$



T_c

2.

$$T_c=\sum_{m=1}^{n}P_m\,I_c(x_m)$$

$$P_1=\frac{1}{2}\,2{,}0\cdot3{,}20=3{,}20\ kN$$

$$P_2=\frac{1}{2}\,2{,}80\cdot3{,}20=4{,}48\ kN$$

$$P_3=\frac{1}{2}\,2{,}80\cdot4{,}80=6{,}72\ kN$$

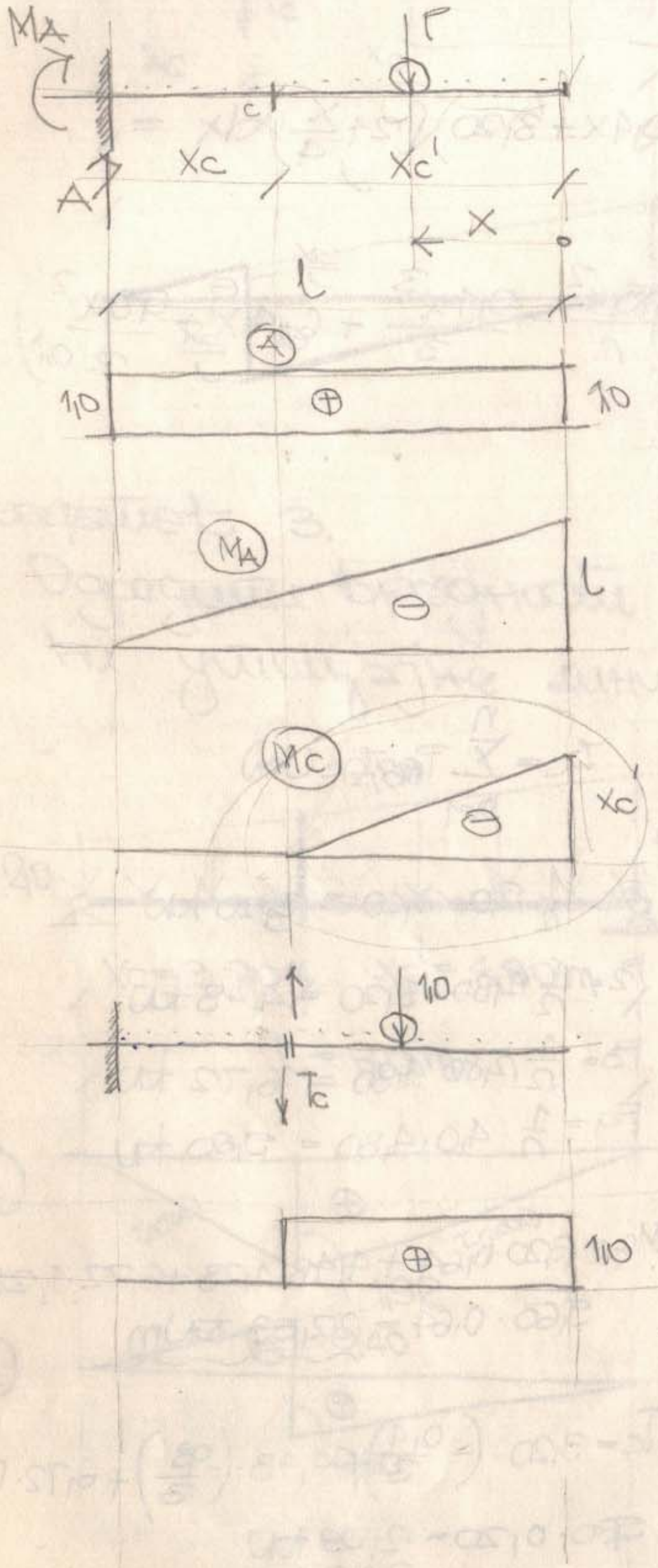
$$P_4=\frac{1}{2}\cdot4{,}0\cdot4{,}80=9{,}60\ kN$$

$$M_c=3{,}20\cdot0{,}64+4{,}48\cdot1{,}28+6{,}72\cdot1{,}28+9{,}60\cdot0{,}64=\underline{22{,}53\ kNm}$$

$$T_c=3{,}20\cdot\left(-\frac{0{,}4}{3}\right)+4{,}48\cdot\left(-\frac{0{,}8}{3}\right)+6{,}72\cdot0{,}40+9{,}60\cdot0{,}20=\underline{2{,}99\ kN}$$

Коментар:
Показано (под 2.) је лакше, рационалније, једноставније, јефтиније,...(али само треба конструисати утицајне линије или утицајну линију).!. (ИММ,2007.)

UTICAJNE LINIJE ZA SILE I REAKCIJE KONZOLE



$\Sigma V = 0 \Rightarrow A - P = 0 \Rightarrow A = P = 1{,}0$

$M_C = -P(x_C' - x) = -(x_C' - x)$

$\boxed{x_C' = l}$ moment u uklještenju

$M_A = -(l - x)$

$x = 0 \Rightarrow M_A = -l$

$x = l \Rightarrow M_A = 0$

za $x = 0 \Rightarrow M_C = -x_C'$ sila desno od presjeka

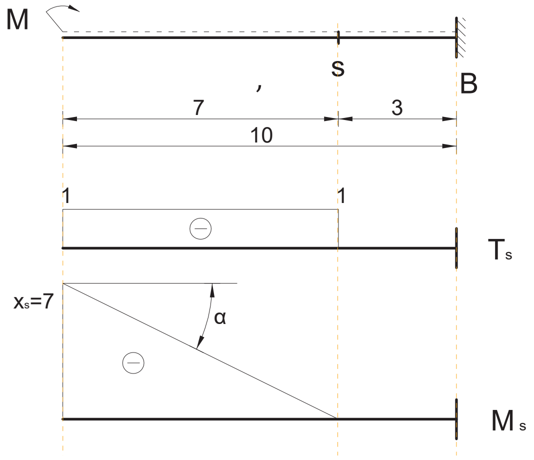
za $x = x_C' \Rightarrow M_C = 0$

$\Sigma V = 0 \Rightarrow T_C - P = 0 \Rightarrow T_C = P = 1{,}0$

$T_1 = -P = -1$

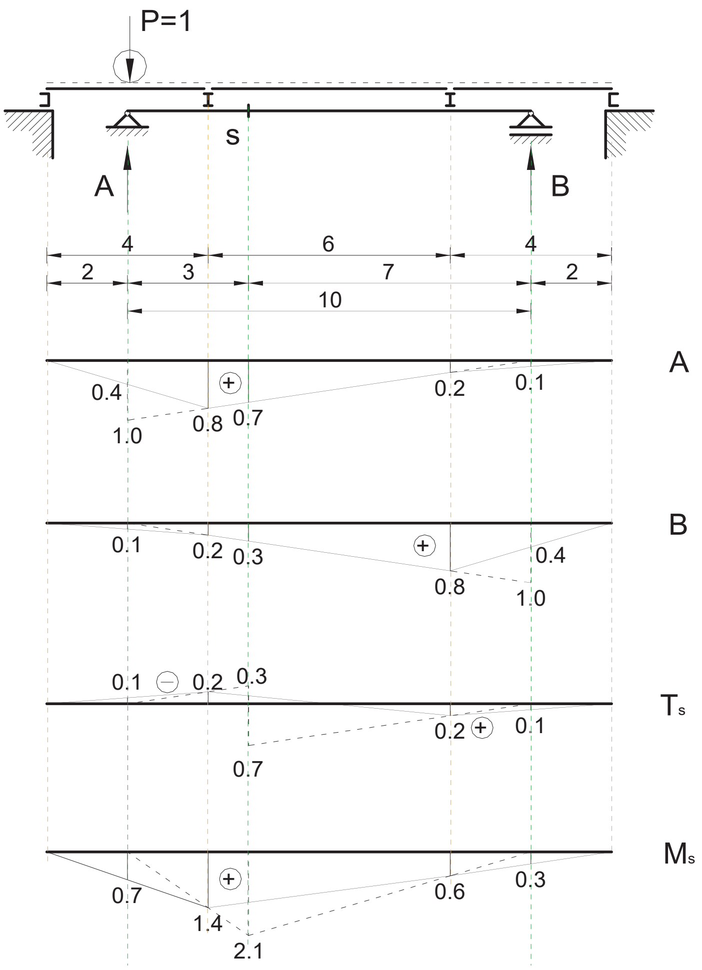
Примери

1. Конструисати утицајне линије за пресечне силе у пресеку „s“ и срачунати вредности утицаја услед деловања $M = 10kNm$ за носач приказан на скици.

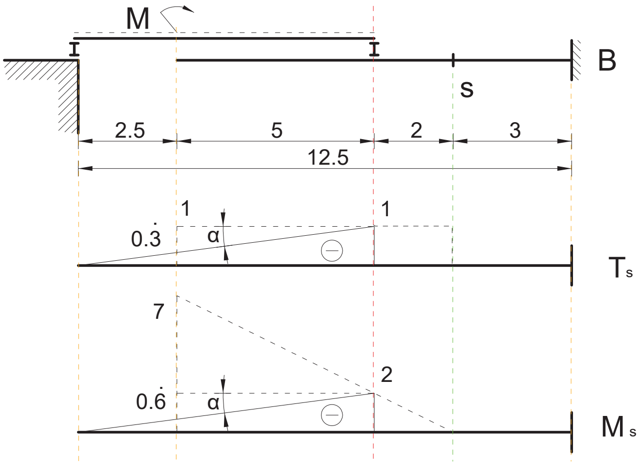


$T_s = M \cdot tan(\alpha) = 10 \cdot (0) = 0$
$M_s = M \cdot tan(\alpha) = 10 \cdot (1) = 10$

2. Конструисати утицајне линије за реакције ослонаца и пресечне силе у пресеку „s“ посредно оптерећене просту греду према скици.

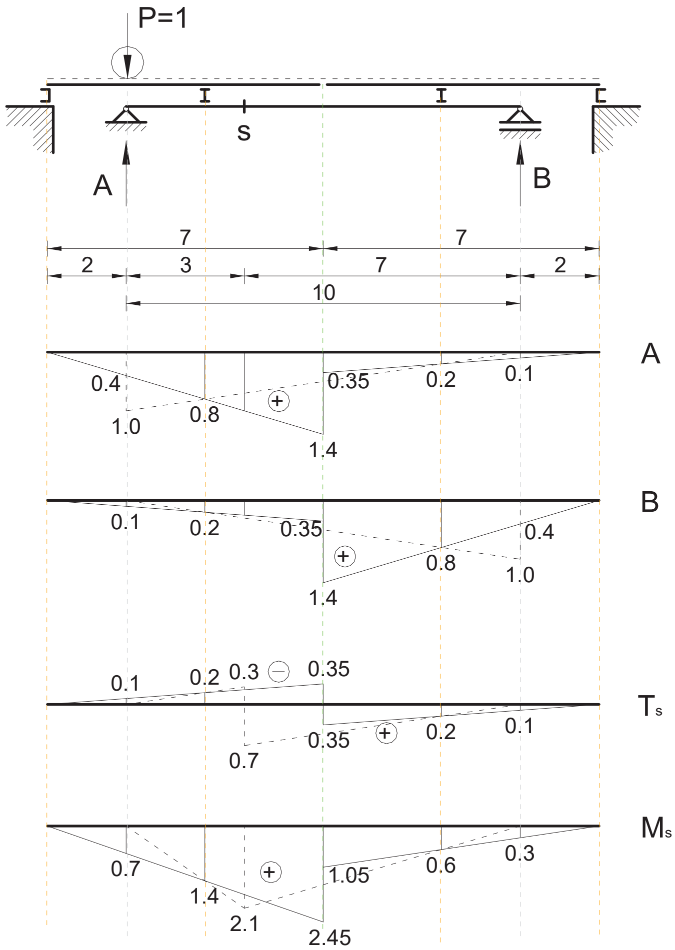


3. Конструисати утицајне линије за пресечне силе у пресеку „s“ и срачунати вредности утицаја услед деловања $M = 10kNm$ за носач приказан на скици.

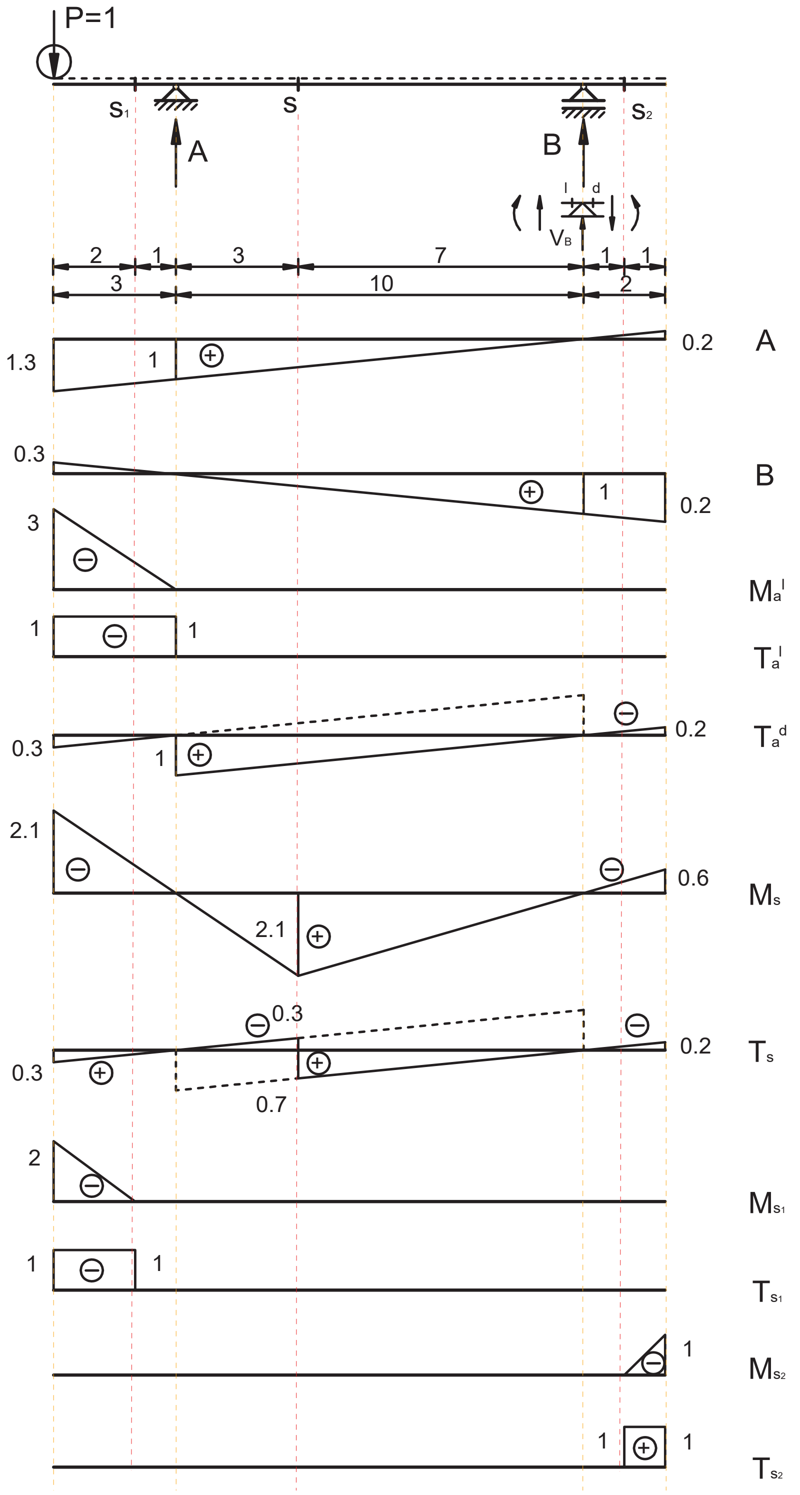


$T_s = M \cdot tan(\alpha) = 10 \cdot (-\frac{0.\dot{3}}{5}) = 10 \cdot (-0.133333) = -1.\dot{3},$
$M_s = M \cdot tan(\alpha) = 10 \cdot (-\frac{0.\dot{6}}{5}) = 10 \cdot (-0.266666) = -2.\dot{6}$

4. Конструисати утицајне линије за реакције ослонаца и пресечне силе у пресеку „s“ посредно оптерећене просту греду према скици.



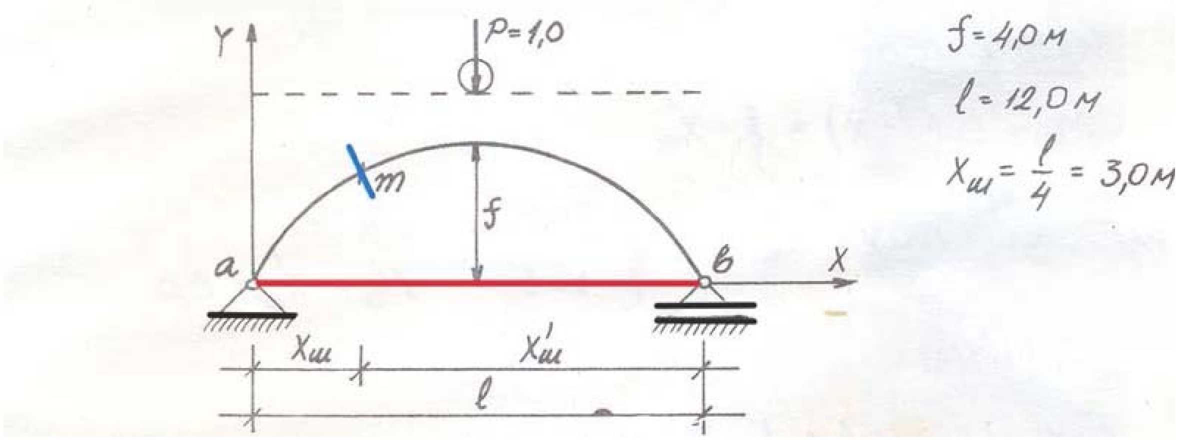
5. Конструисати утицајне линије за реакције ослонаца и пресечне силе у пресецима (s, s_1, s_2) за носач приказан на скици.



Пример

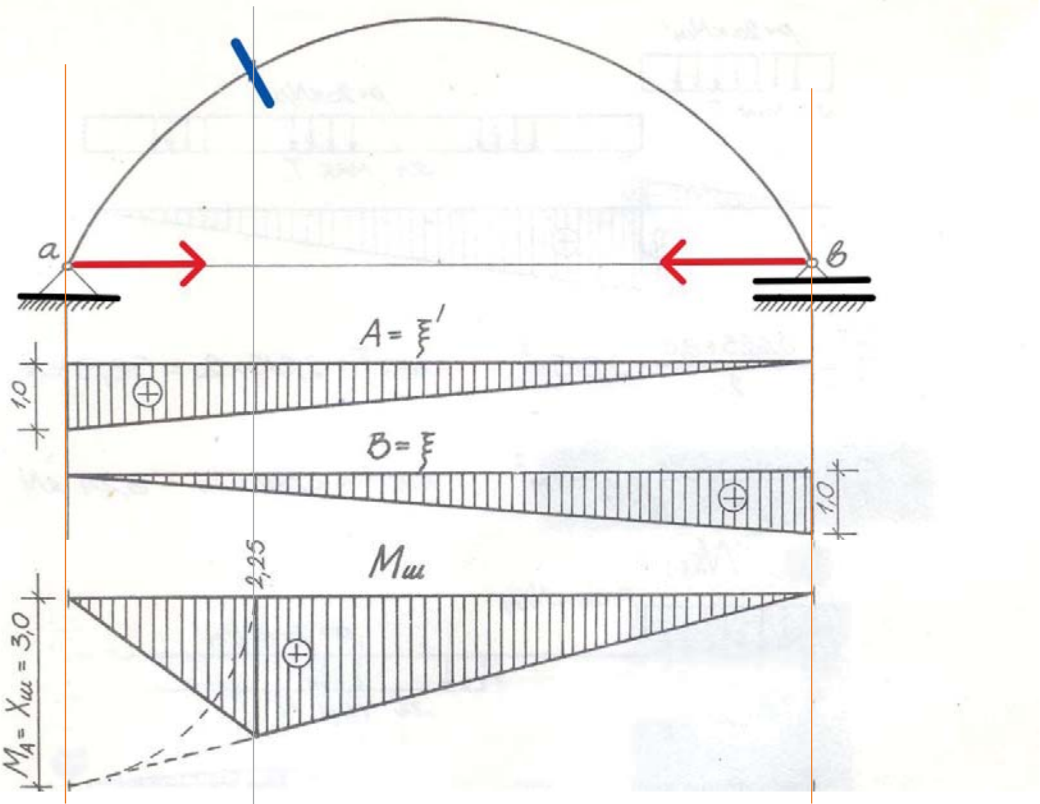
За дати носача приказан на скици:

- нацртати утицајне линије за реакције у ослонцима и силе у пресеку „m“
- услед задатог покретног оптерећења p=20 kN/m, наћи максималне вредности за Va=A, Vb=B, Mm, Tm, Nm.



OSA GREDE IMA JEDNAČINU KVADRATNE PARABOLE :

$$Y = \frac{4f}{l^2}\,x(l-x) = 4f\,\xi\,\xi'$$



Све три утицајне линије имају исти предзнак, тако да максималне позитивне вредности утицаја налазимо према:

$$Z_S = F \times p$$

Према томе, површина утицајне линије за реакцију А, односно В:

$$F_A = 1{,}0\ \frac{12{,}0}{2} = 6{,}0\ m^2$$

max A:

$$max\ A = 6{,}0 \times 20 = 120\ kN$$

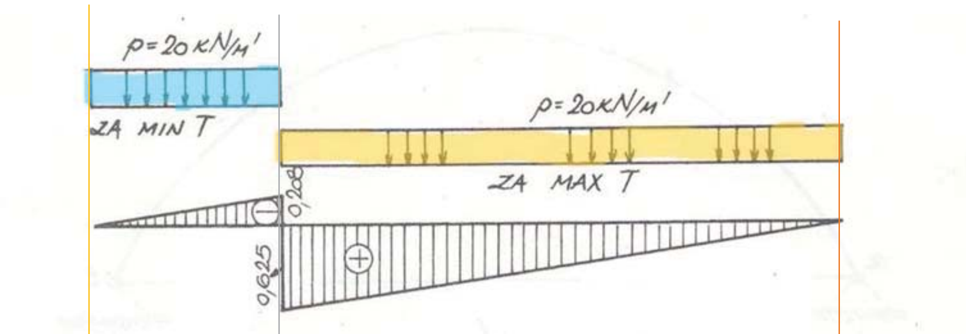
max B:

$$max\ B = 6{,}0 \times 20 = 120\ kN$$

max Mm

$$F_M = 2{,}25\ \frac{12{,}0}{2} = 13{,}5\ m^2 \qquad max\ M = 13{,}5 \times 20 = 270\ kNm$$

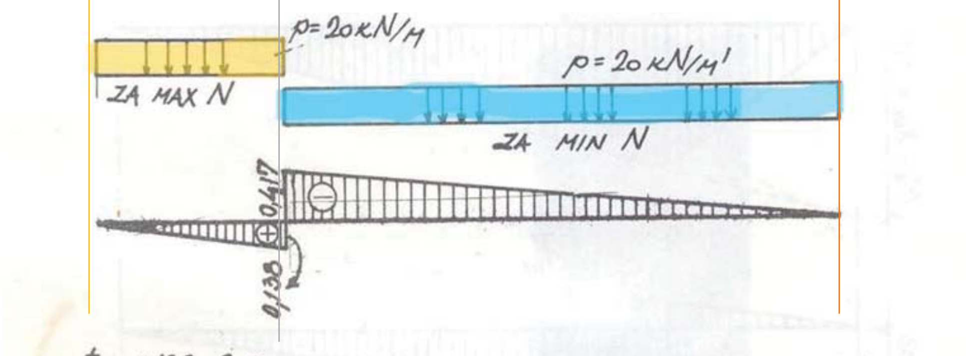
Tm



$$F_T^{+} = \frac{0{,}625 \times 9{,}0}{2} = 2{,}815\ m^2 \qquad max\ T = 2{,}815 \times 20 = 56{,}3\ kN$$

$$F_T^{-} = \frac{0{,}208 \times 3{,}0}{2} = 0{,}312\ m^2 \qquad min\ T = 0{,}312 \times 20 = 6{,}24\ kN$$

Nm



$$F_N^{+} = \frac{0{,}138 \cdot 3{,}0}{2} = 0{,}207\ m^2 \qquad max\ N = 0{,}207 \cdot 20 = 4{,}17\ kN$$

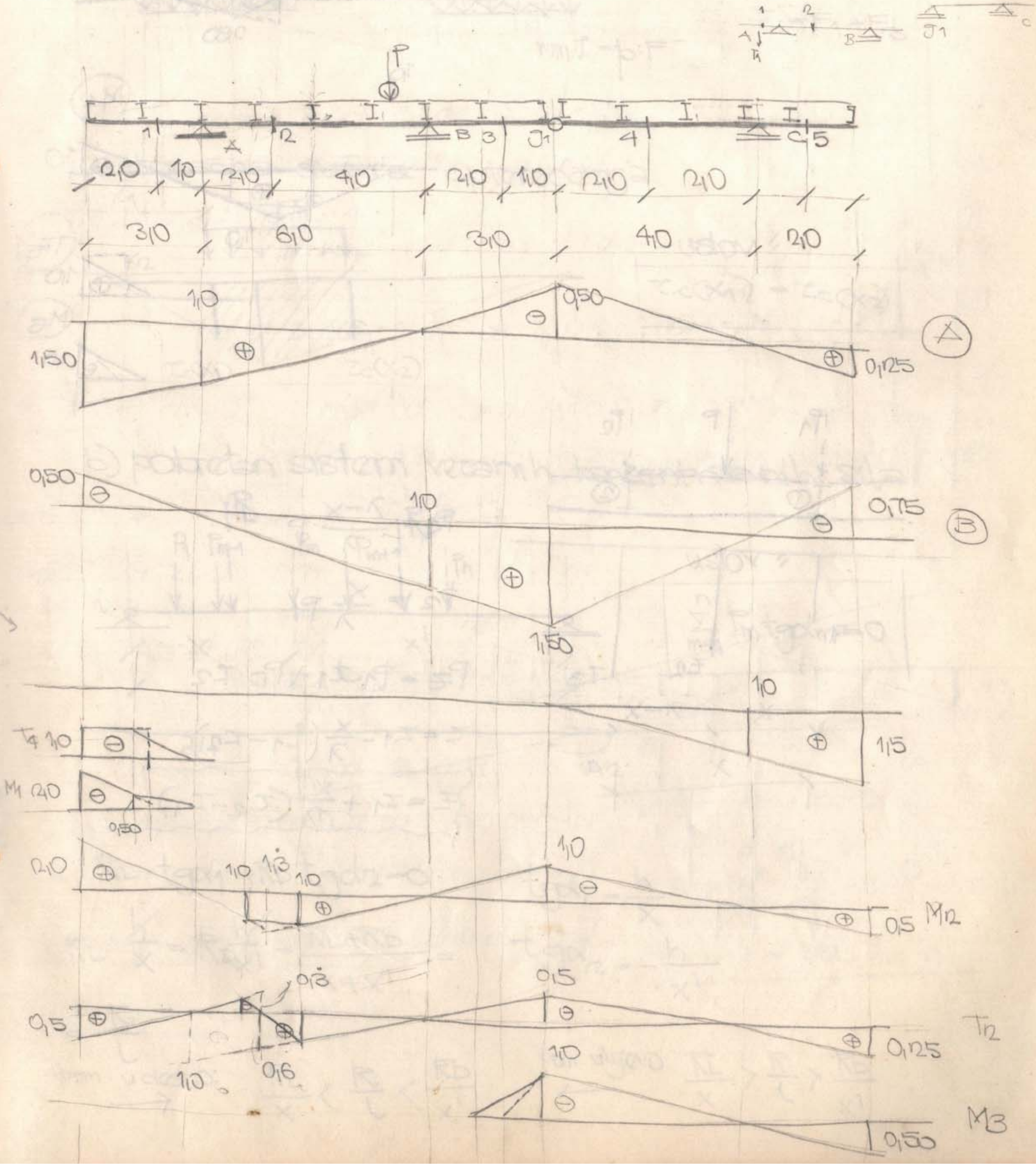
$$F_N^{-} = \frac{0{,}417 \cdot 9{,}0}{2} = 1{,}875\ m^2 \qquad min\ N = 1{,}875 \times 20 = 37{,}5\ kN$$

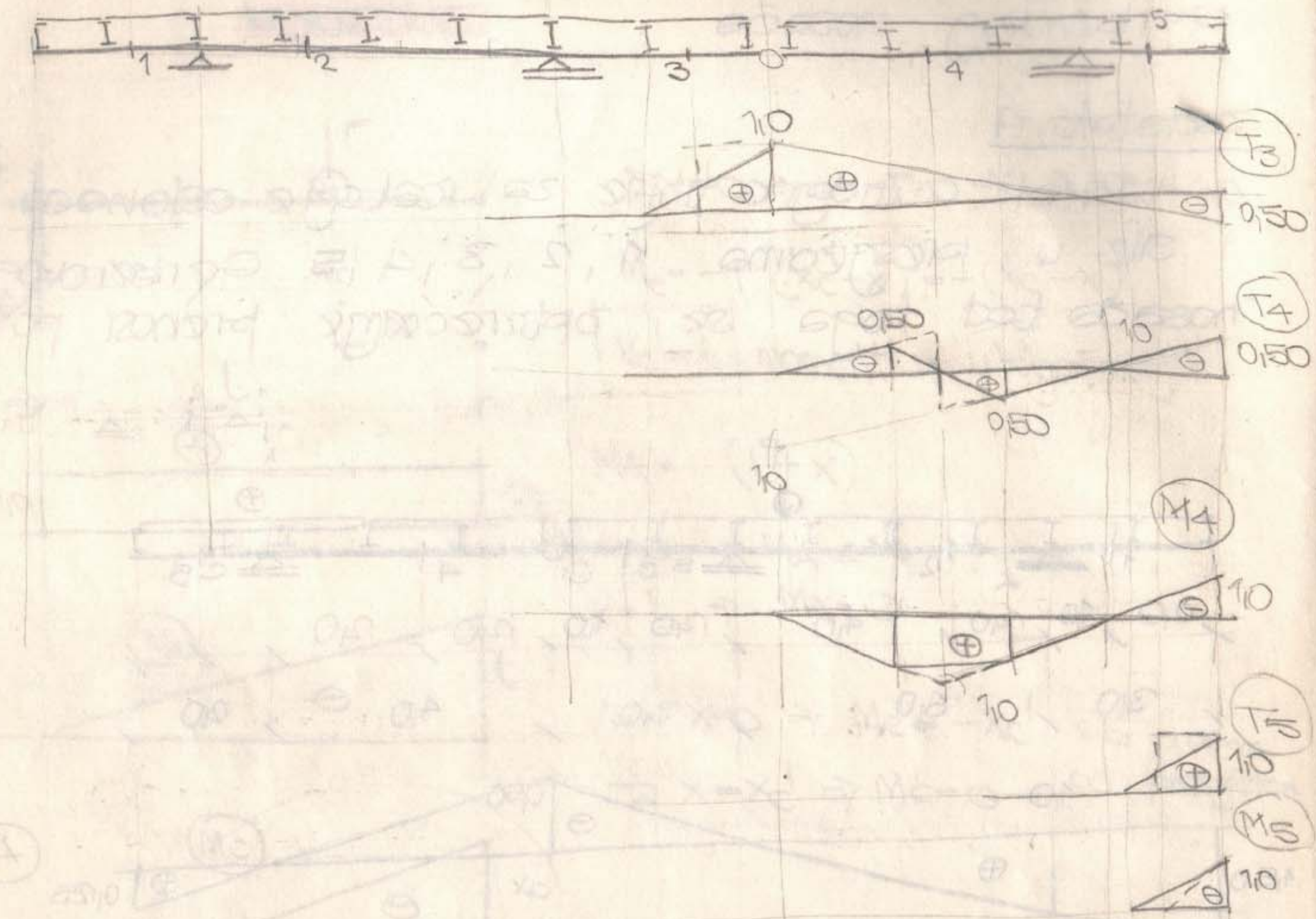
Коментар:
Код третираног носача имамо вертикалне реакције и силу затезања у затеги.

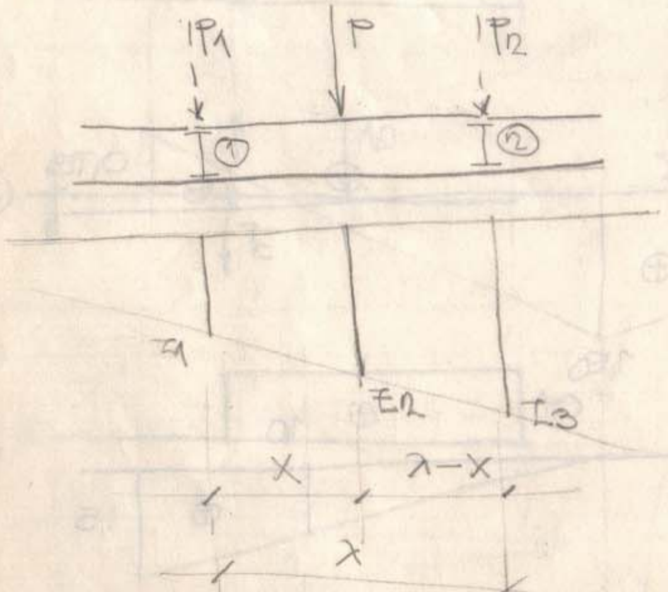
Uticajne linije za reakcije i sile u presjecima Gerberovog nosača

zadatak 4

Nacrtati uticajne linije za reakcije oslonaca i sile u presjecima 1, 2, 3, 4, 5 Gerberovog nosača kod koga se opterećenje prenosi posredno







$$P_1 = \frac{\lambda - x}{\lambda} P$$

$$P_2 = \frac{x}{\lambda} P$$

$$P\tau = P_1 \cdot \tau_1 + P_2 \cdot \tau_2$$

$$\tau = \tau_1 - \frac{x}{\lambda}\left(\tau_1 - \tau_2\right)$$

$$\boxed{\tau = \tau_1 + \frac{x}{\lambda}\left(\tau_2 - \tau_1\right)}$$

Коментар:
Сви приказани задаци руком (ћириличним или латиничним писмом, ијекавским или екавским нарјечјем) писани на остарелом папиру "пожутелим листовима"не датирају из трећег века п.н.е., него из 1991. године.!. (ИММ, 2024.)

Због чега, и како раде у свету на предложену тему…?…

Live Loads for Bridges

- In our previous discussions we mentioned that the primary live loads on bridge spans are due to traffic.
- The heaviest loads are those produced by large transport trucks.
- The American Association of State and Highway Transportation Officials (AASHTO) has a series of specifications for truck loadings.

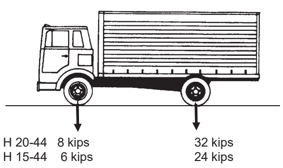
Live Loads for Bridges

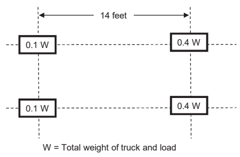
- For two-axial trucks AASHTO designates these vehicles as H series trucks.
- For example, a H15-44 is a 15-ton truck as reported in the 1944 specifications.
- Trucks that pull trailers are designated as HS, for example HS 20-44 (a 20-ton semi-trailer truck).
- In general, a truck loading depends on the type of bridge, its location, and the type of traffic anticipated.

Због чега, и како раде у свету на предложену тему…?… ☺☺☺

Live Loads for Bridges

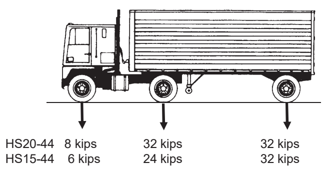
- The size of the "*standard truck*" and the distribution of its weight is reported in the AASHTO code.
- The "H" loading consists of two-axial truck
- The number following the H designation is the gross weight in tons of the standard truck





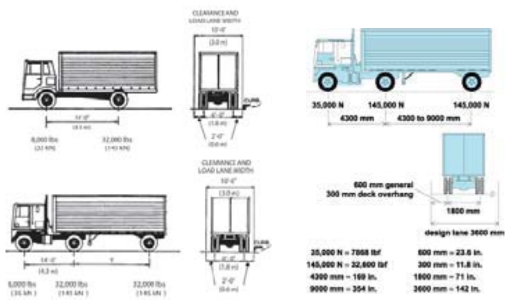
Live Loads for Bridges

- The "HS" loading consists of tractor truck with semi-trailer
- The number following the HS designation is the gross weight in tons of the standard truck



Live Loads for Bridges

The AASHTO standard H20 and HS20 trucks



Live Loads for Bridges

The AASHTO specifications also allow you to represent the truck as a single concentrated load and an uniform load.

For H20-44 and HS20-44:

- Concentrated load 18 kips for moment
  26 kips for shear
- Uniform loading 640 lb/ft of load lane

Live Loads for Bridges

The AASHTO specifications also allow you to represent the truck as a single concentrated load and an uniform load.

For H15-44 and HS15-44:

- Concentrated load 13.5 kips for moment
  19.5 kips for shear
- Uniform loading 480 lb/ft of load lane

Live Loads for Bridges

- You can probably see that once the loading has been selected, you have to determine the critical position of the truck on the structure (bridge).
- This is an excellent application for ***influence lines***.

Live Loads for Bridges

- In many cases, vehicles may bounce or sway as they move over a bridge.
- This motion produces an *impact* load on the bridge.
- AASHTO has develop an ***impact factor*** to increase the live load to account for the bounce and sway of vehicles.

$$I = \frac{50}{L + 125} \leq 0.3$$

where L is the length of the span in feet

Live Loads for Bridges

Impact loading is intended to transfer loads from the superstructure to the substructure

- Superstructures including legs of rigid frames
- Piers excluding footings and those portions below ground line
- Portions above ground line of concrete and steel piles that support the super structure

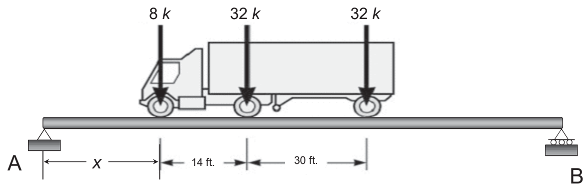
Live Loads for Bridges

Impact shall not be included in loads transferred to footings or to those parts of piles or columns that are below ground

- Abutments, retaining walls, piles excepts as specified before
- Foundation pressures and footings
- Timber structures
- Sidewalk loads
- Culverts and structures having 3 feet or more of cover

Live Loads for Bridges

Example: Consider our standard AASHTO HS20-44 truck traveling over the span of some structure.

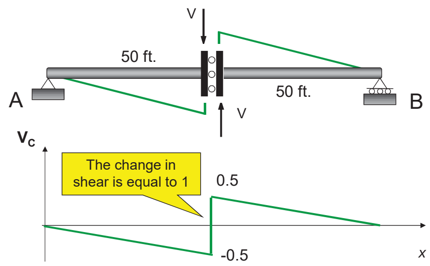


Live Loads for Bridges

- ***Shear*** - To examine how a series of concentrated loads effect the shear lets consider our “standard truck” and its effect on the shear at point C on the beam shown above.
- First we need the influence line for the shear at point C.

Live Loads for Bridges

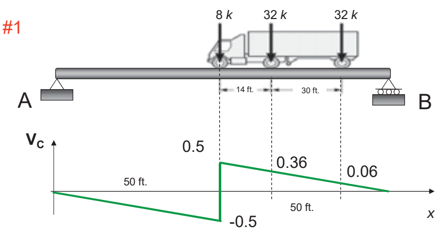
Using the Muller-Breslau principle construct the influence line for the shear at point C



Live Loads for Bridges

- Let’s try to find the maximum *positive* shear at point *C*.
- There are three cases to examine, one for each of the three wheel forces as they pass over the point *C* .

Case #1

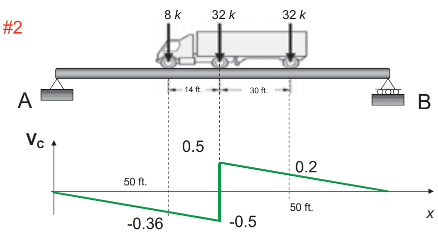


$$(V_C)_{Case\ 1} = 8k(0.5) + 32k(0.36) + 32k(0.06) \quad = 17.44k$$

Live Loads for Bridges

- Let’s try to find the maximum *positive* shear at point *C*.
- There are three cases to examine, one for each of the three wheel forces as they pass over the point *C* .

Case #2

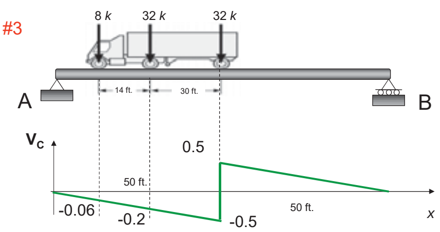


$$(V_C)_{Case\ 2} = 8k(-0.36) + 32k(0.5) + 32k(0.2) \quad = 19.52k$$

Live Loads for Bridges

- Let’s try to find the maximum *positive* shear at point *C*.
- There are three cases to examine, one for each of the three wheel forces as they pass over the point *C* .

Case #3



$$(V_C)_{Case\ 3} = 8k(-0.06) + 32k(-0.2) + 32k(0.5) \quad = 9.12k$$

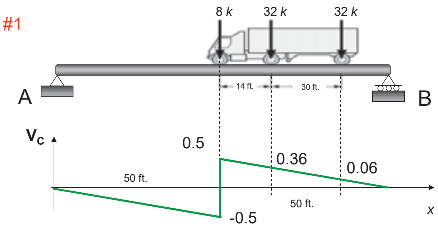
Live Loads for Bridges

- The maximum positive shear at point C is 19.52k
- Let’s rework the previous problem to find the maximum ***negative*** shear at point C.
- There are three cases to examine, one for each of the three wheel forces as they pass over the point C.
- In this case, use the largest ***negative*** value from the influence line

Live Loads for Bridges

- Let's try to find the maximum *negative* shear at point *C*.
- There are three cases to examine, one for each of the three wheel forces as they pass over the point *C* .

Case #1

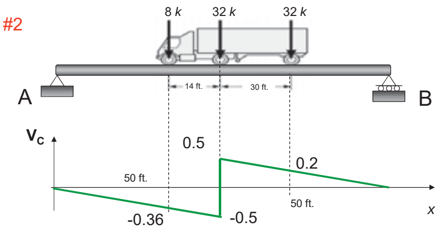


$$(V_C)_{Case\ 1} = 8k(-0.5) + 32k(0.36) + 32k(0.06) \quad = 9.44k$$

Live Loads for Bridges

- Let's try to find the maximum *negative* shear at point *C*.
- There are three cases to examine, one for each of the three wheel forces as they pass over the point *C* .

Case #2

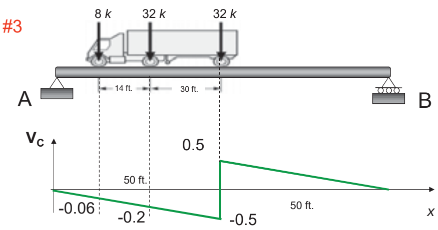


$$(V_C)_{Case\ 2} = 8k(-0.36) + 32k(-0.5) + 32k(0.2) \quad = -12.48k$$

Live Loads for Bridges

- Let's try to find the maximum *negative* shear at point *C*.
- There are three cases to examine, one for each of the three wheel forces as they pass over the point *C* .

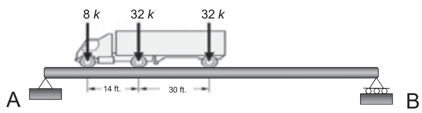
Case #3



$$(V_C)_{Case\ 3} = 8k(-0.06) + 32k(-0.2) + 32k(-0.5) \quad = -22.88k$$

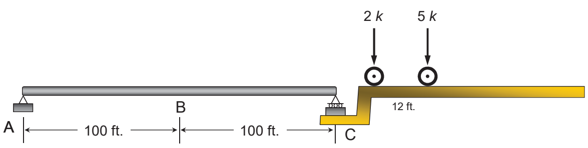
Live Loads for Bridges

- The maximum ***negative*** shear at C is -22.88k
- In this case, the largest shear at C is the largest ***negative*** value, or V_{max} = -22.88k



Live Loads for Bridges

Example: Determine the maximum moment created at point B in the beam below due to the wheel loads of a moving truck. The truck travels from right to left.



Live Loads for Bridges

Example: Determine the maximum shear created at point C in the beam below due to the wheel loads of a moving truck. The truck travels from right to left.