11

СТАТИКА КОНСТРУКЦИЈА

Модул: Хидротехника и водно инжењерство околине, Саобраћајнице, Архитектонско инжењерство

– материјал за вежбе –

2024.

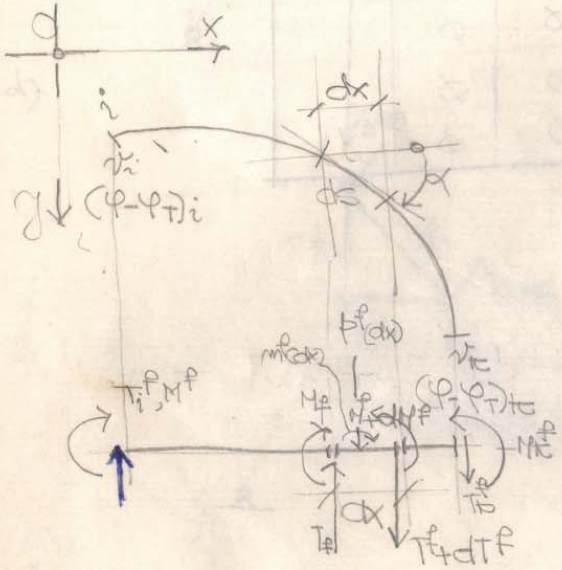
24.04.1991

DIJAGRAM POMERANJA PUNI RAVNI NOSAČA

$$du = \varepsilon dx - \varphi dy$$

$$d(\varphi - \varphi_T) = -\varkappa ds \;/\cdot \cos\alpha \Rightarrow \frac{d(\varphi-\varphi_T)}{dx} = -\frac{\varkappa}{\cos\alpha}$$

$$dv = \varepsilon dy + \varphi dx \;/\cdot dx \Rightarrow \frac{dv}{dx} = \varepsilon \cdot tg\alpha + \varphi \rightarrow \underbrace{\frac{dv}{dx} = (\varphi - \varphi_T) + \varepsilon tg\alpha + \varphi_T}_{\text{(1)}}$$



$$\Sigma V = 0 \;/$$

$$\frac{dT^f}{dx} = -p^f$$

$$\Sigma M_a = 0$$

$$\frac{dM^f}{dx} = T^f + m^f \quad \ldots (2)$$

$$p^f \leftrightarrow \frac{\varkappa}{\cos\alpha}$$

$$m^f \leftrightarrow \varepsilon tg\alpha + \varphi_T$$

$$M^f \leftrightarrow v \;;\; T^f \leftrightarrow (\varphi - \varphi_T)$$

$$p^f = \frac{\varkappa}{\cos\alpha} = \left(\frac{M}{EJ} + \alpha_t \frac{\Delta t}{h}\right)\cdot\frac{1}{\cos\alpha}$$

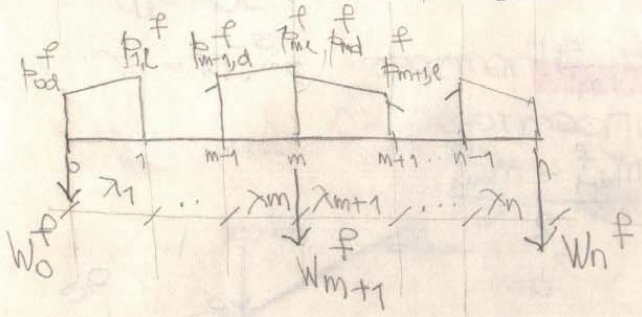
$$m^f = \varepsilon \cdot tg\alpha + \varphi_T = \left(\frac{N}{EF} + \alpha_t t^0\right) tg\alpha + t \cdot \frac{I}{GF}$$

Dijagram vertikalni pomeranja (V) jednak je dijagramu momenata m^f, a dijagram obtanja jednak je dijag. transverzalni sila T^f, fiktivnog štapa opterećenog fiktivnim raspored. silama p^f, i fiktivnim raspod. momentima m^f čiji granični uslovi po silama odgovaraju

graničnim uslovima po pomeranju na stranom nosaču.

W – ELASTIČNE TEČNE

W– a) **Statička promjena opterećenja p^f**



$$W_0^f = \frac{\lambda_1}{6}\left(2p_0^f + p_{1,l}^f\right)$$

$$W_{m+1}^f = \frac{\lambda_m}{6}\left(2p_{m,l}^f + p_{m-1,d}^f\right) + \frac{\lambda_{m+1}}{6}\left(2p_{m,d}^f + p_{m+1,l}^f\right) \quad m-1,2,\dots,n-1$$

$$W_n^f = \frac{\lambda_n}{6}\left(2p_n^f + p_{n-1,d}^f\right)$$

b) **LINEARNA TROMOĐA p^f**

$\lambda = \text{const.}$

$$W_0^f = \frac{\lambda}{6}\left(2p_0^f + p_1^f\right)$$

$$W_m^f = \frac{\lambda}{6}\left(p_{m-1}^f + 4p_m^f + p_{m+1}^f\right) \quad m=1,2,\dots,n-1$$

$$W_n^f = \frac{\lambda}{6}\left(2p_n^f + p_{n-1}^f\right)$$

c) **PARABOLIČNA TROMOĐA p^f**

$$W_0^f = \frac{\lambda}{24}\left(7p_0^f + 6p_1^f - p_2^f\right)$$

$$W_m^f = \frac{\lambda}{12}\left(p_{m-1}^f + 10p_m^f + p_{m+1}^f\right) \quad (m=1,2,\dots,n-1)$$

$$W_n^f = \frac{\lambda}{24}\left(7p_n^f + 6p_{n-1}^f - p_{n-2}^f\right)$$

– SKOKOVITA TRANSLACIJA m^f

$$W_0^{mf} = -\frac{m_0^f + m_{1,\ell}^f}{2} \quad ; \quad W_m^{mf} = \frac{m_{m-1,d}^f + m_{m,\ell}^f}{2} - \frac{m_{m,d}^f + m_{m+1,\ell}^f}{2}$$

$$W_n^{mf} = \frac{m_{n-1,d}^f + m_n^f}{2}$$

– LINEARNA TRANSLACIJA m^f

$$W_0^{mf} = -\frac{m_0^f + m_1^f}{2} \qquad W_m^{mf} = \frac{m_{m-1}^f - m_{m+1}^f}{2}$$

$$W_n^{mf} = \frac{m_{n-1}^f + m_n^f}{2}$$

GRANIČNI USLOVI:

R. BROJ	STVARNI NOSAČ	FIKTIVNI NOSAČ
1.	$v_i = 0$, $(\varphi - \varphi_T)_\ell \neq 0$	$M_i^f = 0$, $T_i^f \neq 0$
2.	$v_i = 0$, $(\varphi - \varphi_T)_i = 0$	$M_i^f = 0$, $T_i^f = 0$
3.	$v_i \neq 0$, $(\varphi - \varphi_T)_i \neq 0$	$M_i^f \neq 0$, $T_i^f \neq 0$
4.	$v_i \neq 0$, $(\varphi - \varphi_T)_\ell = (\varphi - \varphi_T)_d \neq 0$	$M_i^f \neq 0$, $T_\ell^f = T_d^f \neq 0$
5	$v_i \neq 0$, $(\varphi - \varphi_T)_i^\ell \neq (\varphi - \varphi_T)_i^d \neq 0$	$M_i^f \neq 0$, $T_\ell^f \neq T_d^f \neq 0$
6	$v_i = 0$, $(\varphi - \varphi_T)_\ell = (\varphi - \varphi_T)_d \neq 0$	$M_i = 0$, $T_\ell^f = T_d^f \neq 0$
7.	$v_i = 0$, $(\varphi - \varphi_T)_\ell \neq (\varphi - \varphi_T)_\ell \neq 0$	$M^f = 0$, $T_\ell \neq T_d \neq 0$

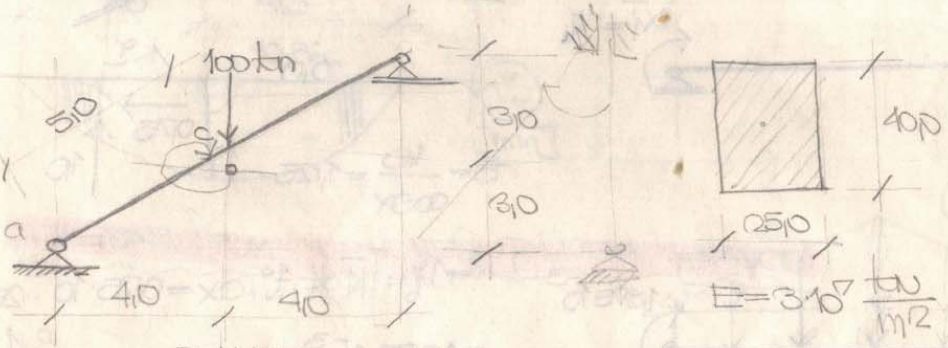
kruta veza treba da postoji

ZADATAK. Odrediti dijagrame pom. za istovremeno djelovanje opterećenja i temperature

$t_0 = 10°C$

$t_u = 40°C$

$\alpha_t = 10^{-5}/°C$, zanemariti utjecaj transferzalnih sila na deformaciju.



PRAVCU

a) POM. U PRESJEKU OSE ŠIPKE

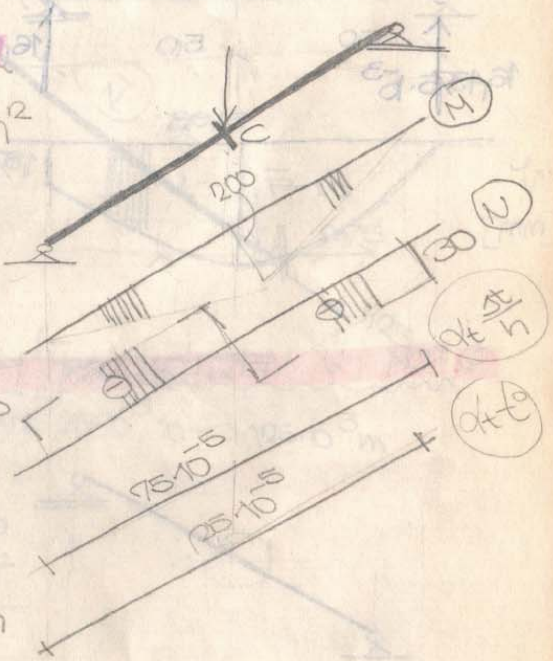
$EJ = 3 \cdot 10^7 \frac{0{,}25 \cdot 0{,}40^3}{12} = 4 \cdot 10^4 \text{ kNm}^2$

$EF = 3 \cdot 10^7 \cdot 0{,}25 \cdot 0{,}40 = 3 \cdot 10^6 \text{ kN}$

$\Delta L = \frac{S \cdot L}{EF} + \alpha_t \cdot t^\circ \cdot L$ — ИЗАЖЕЊЕ У ОСИ ШТАПА

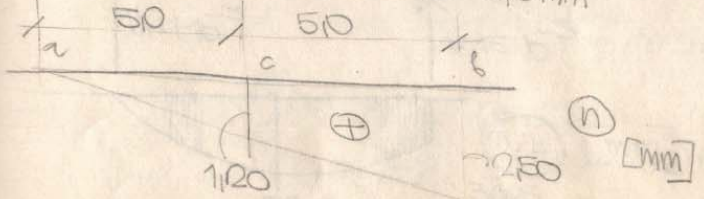
$\Delta t^\circ = t_u - t_0 = 30°C$

$t^\circ = \frac{t_u + t_0}{2} = 25°C$ temp. u osi štapa



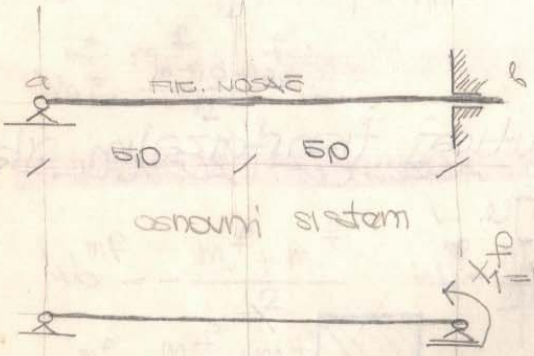
$\Delta L_{2c} = \frac{-30 \cdot 5}{3 \cdot 10^6} + 25 \cdot 10^{-5} \cdot 5 = 1{,}20 \cdot 10^{-3} \text{ m} = 1{,}2 \text{ mm}$

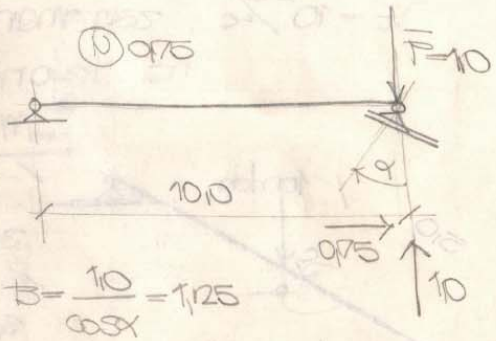
$\Delta L_{a,b} = 25 \cdot 10^{-5} \cdot 10 = 25 \cdot 10^{-3} \text{ m} = 2{,}5 \text{ mm}$



b) POM. UTJECAJ NA OSU ŠTAPA

$\alpha = 0$

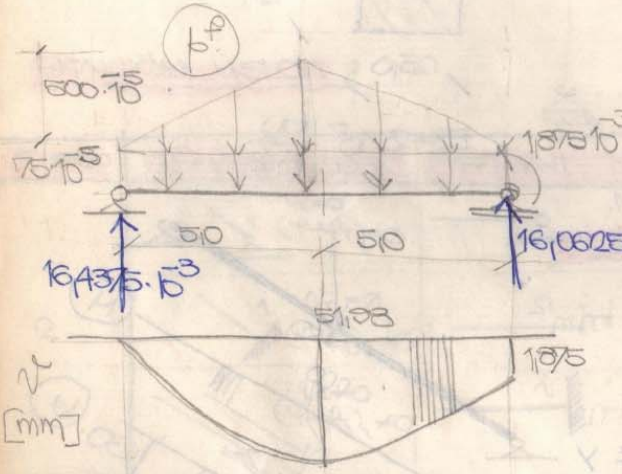




$$\cos\alpha = \frac{1,0}{0,8} = 1,25$$

$$X_1^p = v_b = \int \bar{N}\alpha_t t^\circ \cdot dx = 0,75 \cdot 10^{-5} \cdot 25 \cdot 10,0 =$$

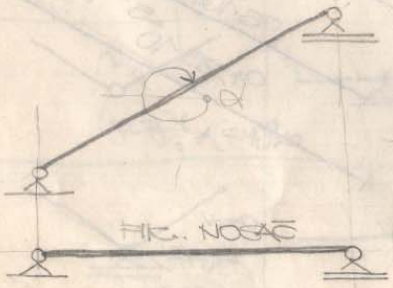
$$= 1,875 \cdot 10^{-3}\,m$$



$$b_t^p = \left(\frac{M}{EJ} + \alpha_t \frac{\Delta t}{h}\right)\frac{1}{\cos\alpha}\,dx$$

$$N_t^p = \left(\frac{N}{EF} + \alpha_t t^\circ\right)\frac{1,0}{\cos\alpha}$$

c) POM. U VERTIKALNOM PRAVCU

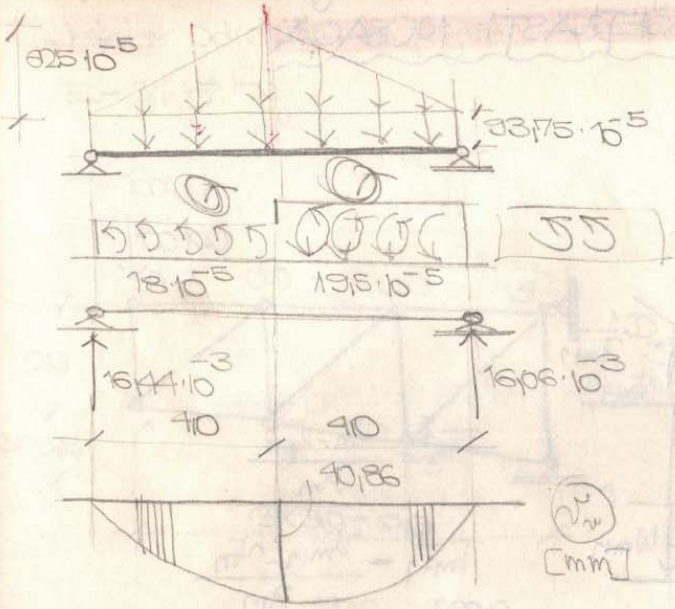


$$\sin\alpha = -0,6$$

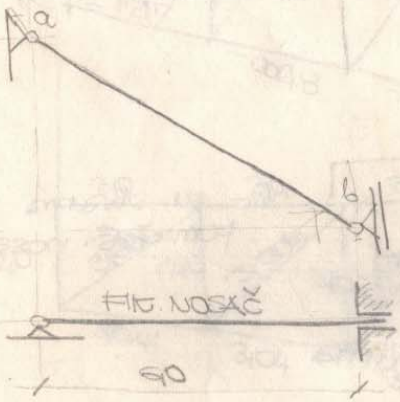
$$\cos\alpha = 0,8$$

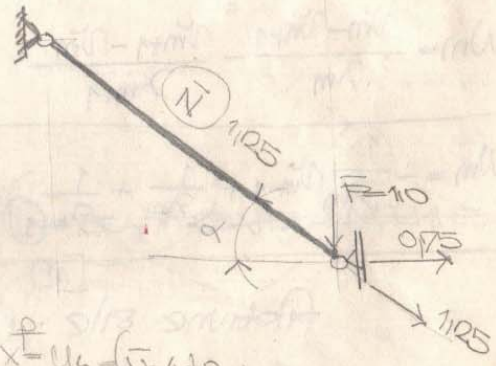
$$\operatorname{tg}\alpha = -0,75$$

$$b_t^p = \left(\frac{M}{EJ} + \alpha_t \cdot \frac{\Delta t}{h}\right)\frac{1}{\cos\alpha}$$



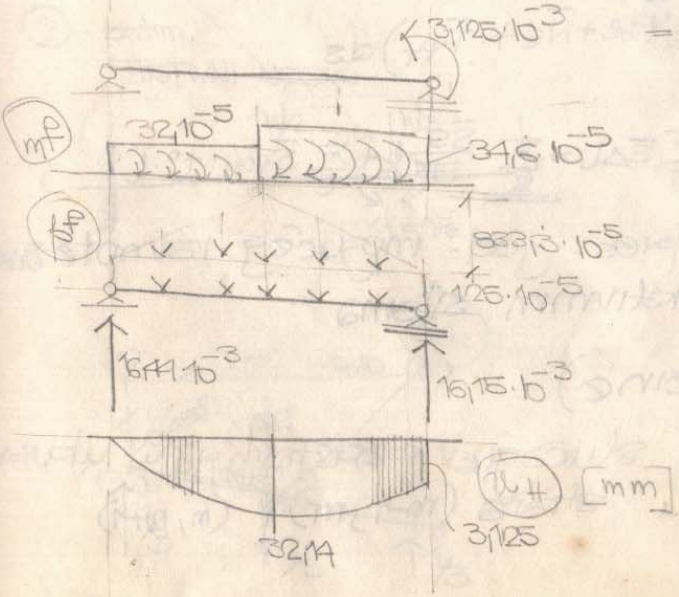
d) DIJAGRAM POM. U HORIZONTALNOM PRAVCU



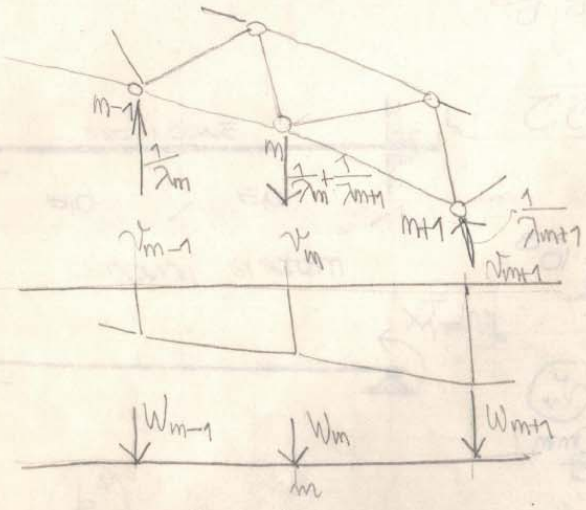


$$x = u_b = \int \bar{N} \cdot t^\circ \cdot dx =$$

$$= 1{,}25 \cdot 25 \cdot 10^{-5} \cdot 10 = 3{,}125 \cdot 10^{-3}\ m$$

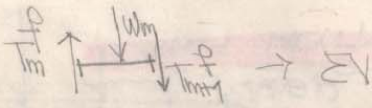


DIJAGRAM TOM. REŠETKASTI NOSAČA



$$T_m^f = \frac{\bar{v}_m - \bar{v}_{m-1}}{\lambda_m}$$

$$T_{m+1}^f = \frac{\bar{v}_{m+1} - \bar{v}_m}{\lambda_m}$$



$$\rightarrow \Sigma Y = 0 \Rightarrow W_m = T_m^f - T_{m+1}^f$$

$$W_m = \frac{\bar{v}_m - \bar{v}_{m-1}}{\lambda_m} - \frac{\bar{v}_{m+1} - \bar{v}_m}{\lambda_{m+1}}$$

$$W_m = -\frac{1}{\lambda_m}\bar{v}_{m-1} + \left(\frac{1}{\lambda_m} + \frac{1}{\lambda_{m+1}}\right)\bar{v}_m - \frac{1}{\lambda_{m+1}}$$

$\bar{v}_m$ - su stvarna pom. reš. nosača

fiktivne sile u čvorovima

- PUNE NOSAČE: $W_m = \int_S (\bar{M}\varkappa + \bar{N}\varepsilon + \bar{T}\varphi_T)\,ds$

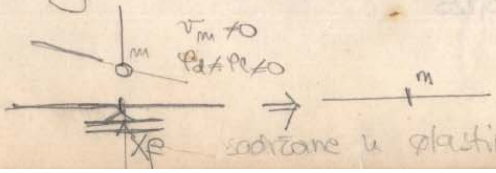
REŠETKAST NOSAČ:

$$W_m = \sum_S \bar{S}\Delta L = \sum_S \frac{\bar{S}S}{EA} L + \sum_S \bar{S}\cdot\alpha_t\cdot t^\circ\cdot L$$

- $\bar{M}; \bar{N}; \bar{T}; \bar{S}$; - sile u pres. jed. mogućeg ravnotežnog stanja opterećenja fiktivnim silama

- ($\varkappa, \varepsilon, \varphi_T$ - def. veličine)

- El. težine u ovom slučaju predstavljaju ukupne promjene ugla između strana $(m-1, m)$ i $(m, m+1)$

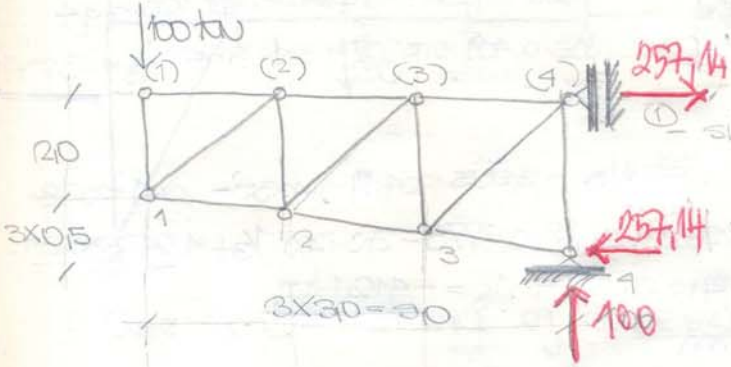


$\Rightarrow$ sadržane u plastičnim težinama

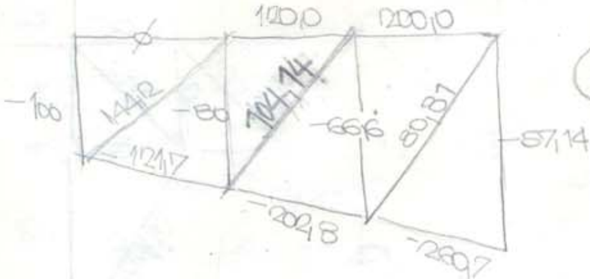
zadatak: odrediti dijagram pom. donjeg pojasa reš. nosača

$E = 2{,}1\cdot 10^{8}\ \text{tou}/m^2$

$F = 100\ cm^2$

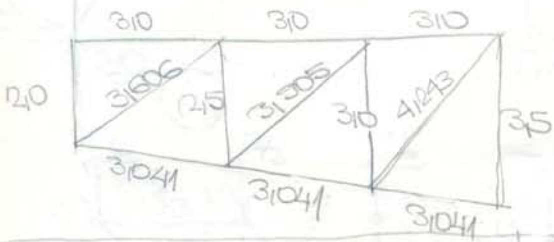


① – sile u štap od opterećenja



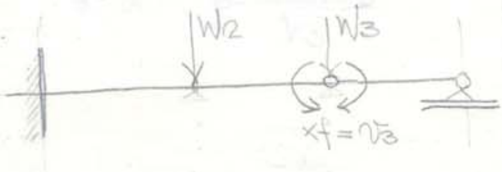
S [kN]

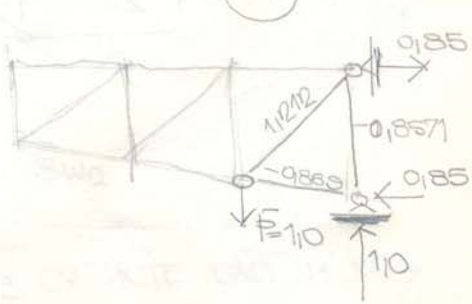
$l' = \frac{F_c}{F}\, l$

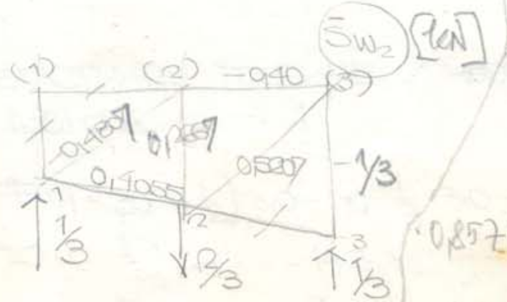


l' – РЕДУКОВАНЕ ДУЖИНЕ [m]

② – zatim,
FIKTIVNI NOSAČ



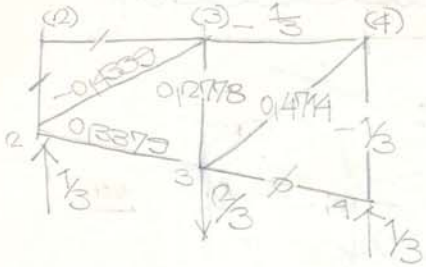




$EF v_3 = \sum_S S \bar{S}_v \cdot l$

$EF v_3 = 3{,}041\cdot 289{,}7\cdot 0{,}8680 + 3{,}50\cdot 57{,}14\cdot 0{,}8571 + 4{,}243\cdot 80{,}81\cdot 1{,}212 = 1276\ \text{tovm}$

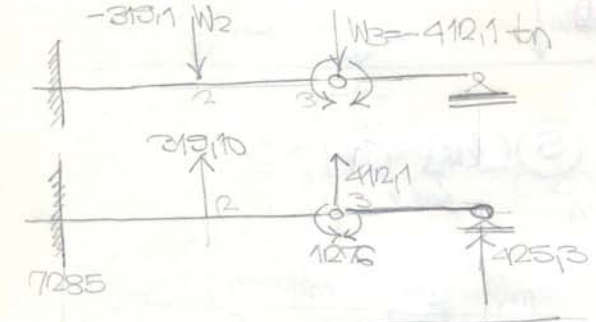
$v_3 = \frac{1276}{\frac{100}{10000}\cdot 2{,}1\cdot 10^{8}} = 0{,}000608\ m = 0{,}608\ mm$

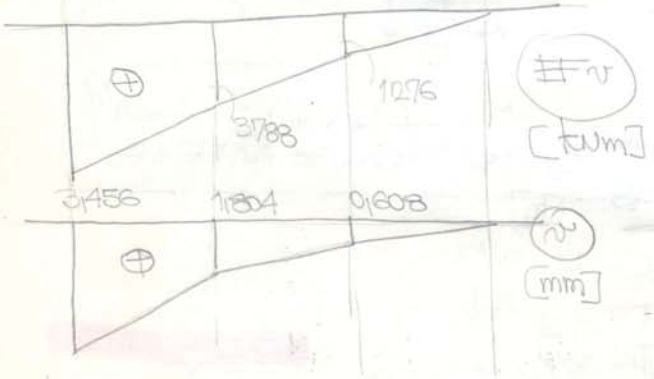


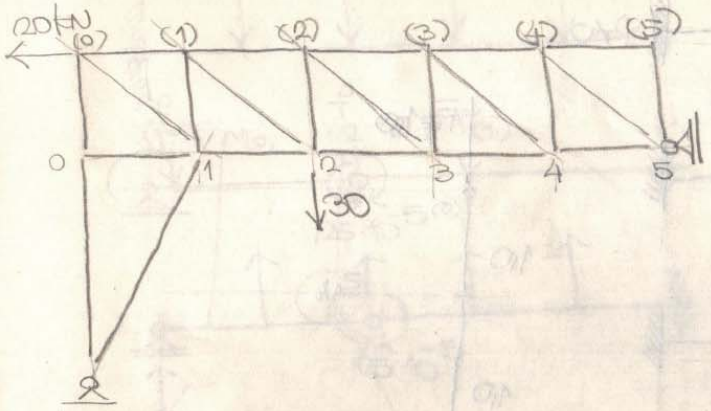
ΣW_3

$\Sigma F W_2 = -3{,}606 \cdot 144{,}12 \cdot 0{,}4807 - 3{,}041 \cdot 121{,}17 \cdot 0{,}4055 - 2{,}80 \cdot 80 \cdot 0{,}2667 - 3{,}0 \cdot 120 \cdot 0{,}44 + 3{,}205 \cdot 0{,}401 \cdot 0{,}5207 + 3{,}0 \cdot 666 \cdot \frac{1}{3} = -319{,}1$ ton

$\Sigma F W_3 = -3{,}605 \cdot 104{,}14 \cdot 0{,}4833 - 3{,}041 \cdot 202{,}8 \cdot 0{,}33373 - 3{,}0 \cdot 66{,}6 \cdot 0{,}12778 - 3{,}0 \cdot 200 \cdot \frac{1}{3} + 4{,}243 \cdot 80{,}81 \cdot 0{,}47714 + 3{,}5 \cdot 87{,}14 \cdot \frac{1}{3} = -412{,}1$ ton





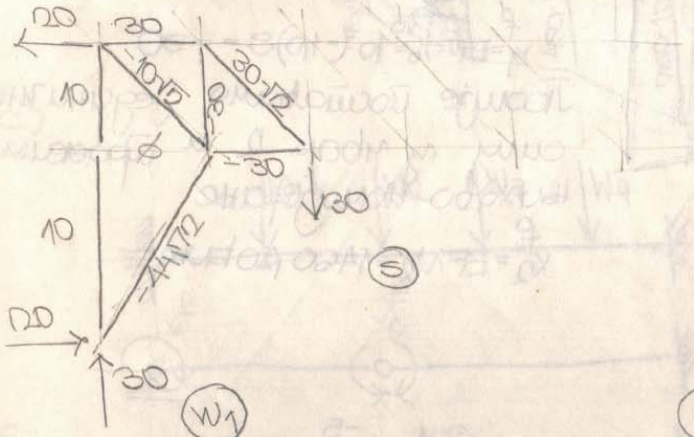


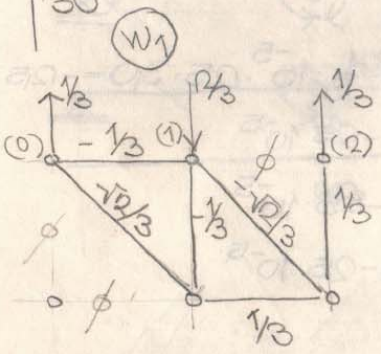
$t° = 25cm$

$EF = const.$

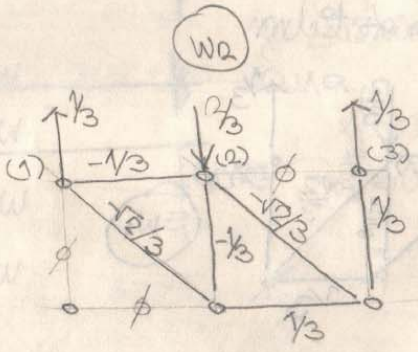
$$W = \sum_m \frac{S_m \cdot \bar{S}_m \cdot l_m}{EF_m} + \sum \bar{S}_m \cdot \alpha_t \cdot t° \cdot l_m$$

а) Силе у штаповима од спољног оптерећења





$\bar{S}_{W_1}$



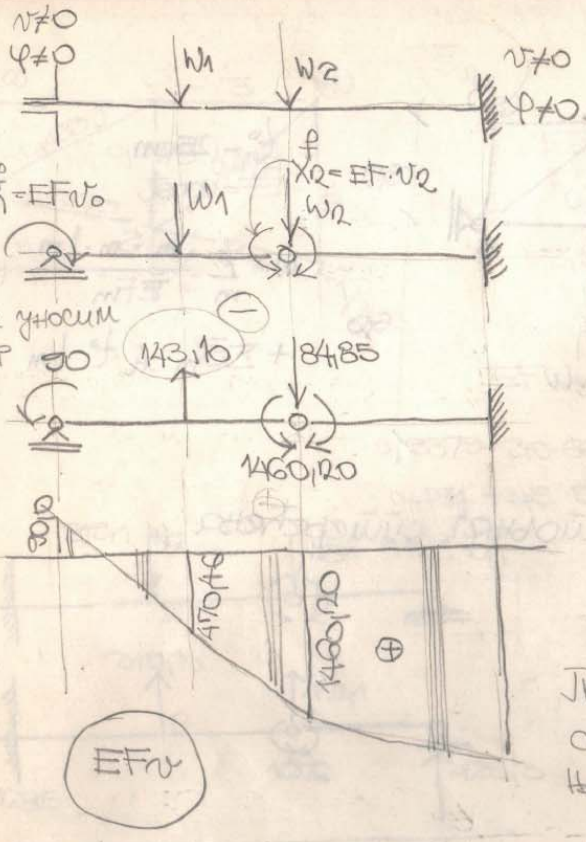
$\bar{S}_{W_2}$

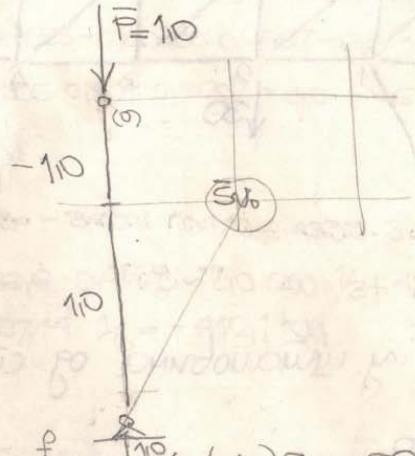
– ЕКАСТИЧНЕ ТЕЖИНЕ У ЧВОРУ 4 И 3 СУ ИСТЕ ЕКО И КОД ЧВОРОВА 2 И 1.

$$EF_{W_1} = 30 \cdot \left(-\frac{1}{3}\right) 310 - 30 \cdot \frac{1}{3} \cdot 310 - 10\sqrt{2} \cdot \frac{\sqrt{2}}{3} \cdot 30\sqrt{2} + 30 \cdot \sqrt{2} \left(-\frac{\sqrt{2}}{3}\right) 310 -$$

$$- 30 \cdot \left(-\frac{1}{3}\right) \cdot 310 = -143{,}10$$

$$EF_{W_2} = 30\sqrt{2} \cdot \frac{\sqrt{2}}{3} \cdot 30\sqrt{2} = 84{,}85$$





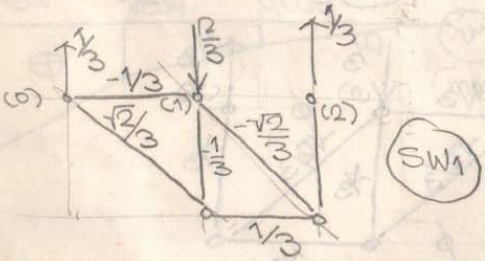
$X_1^f = EFv_0 = 10 \cdot (-1{,}0) \cdot 9 = -90$

Тамније постављамо јединичну силу у чвор 2 и цртамо њихово померање

$X_2^f = EFv_2 = 1460{,}20\ kNm$

B) $t° = 25°C$

$w_m = \sum \bar{S}_m \cdot \alpha t \cdot t° \cdot l_m$

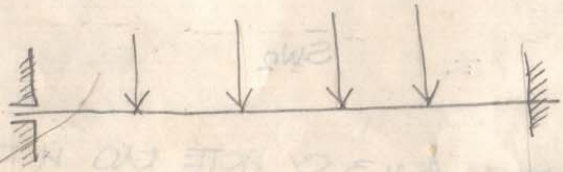


$w_1 = -\frac{1}{3} \cdot 10^{-5} \cdot 25 \cdot 30 = -25 \cdot 10^{-5}$

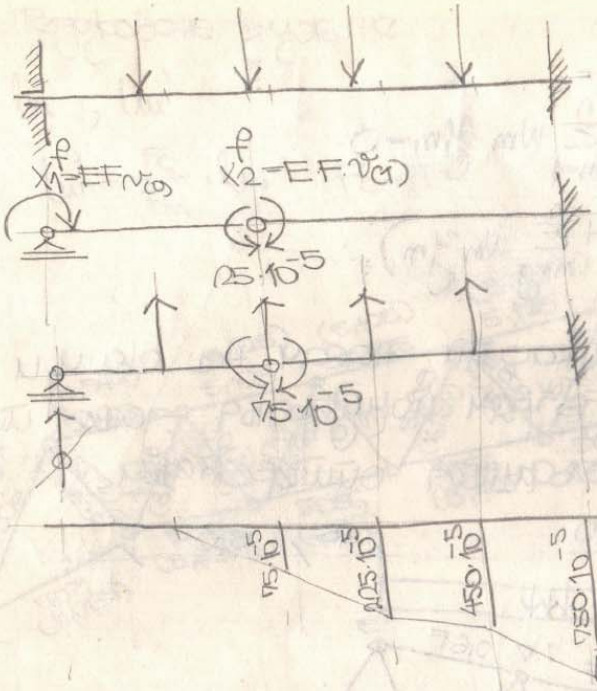
$w_2 = -25 \cdot 10^{-5}$

$w_3 = -25 \cdot 10^{-5}$

$w_4 = -25 \cdot 10^{-5}$



r

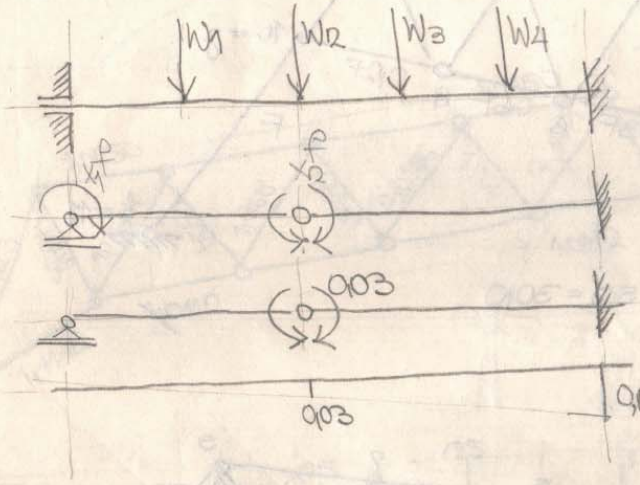


$X_1^p = EF\bar{v}_{(0)} = 0$

због тога што услед јединичне силе у чвору (c) нема сила у штапу горњег појаса

$X_2^p = EF\bar{v}_{(1)} - 75 \cdot 10^{-5}$

c)

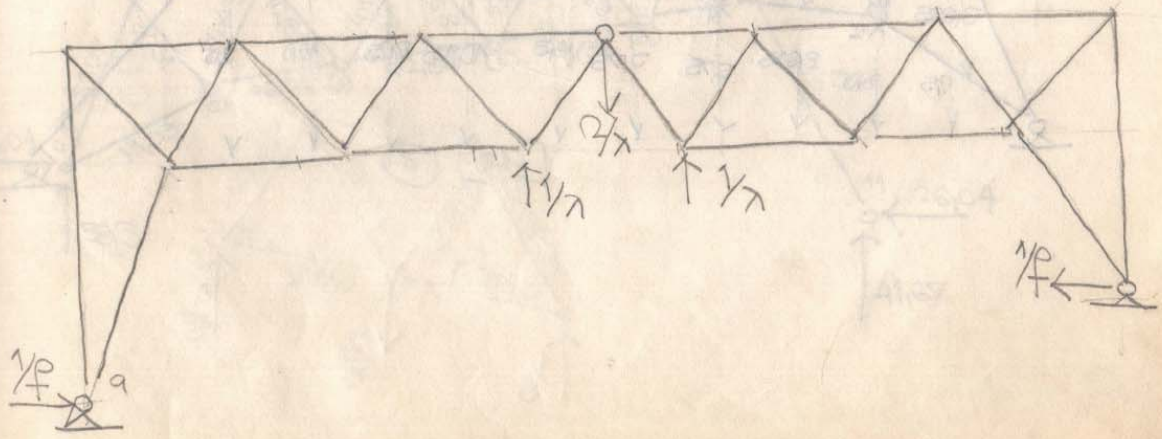


све су еластичне деформације једнаке су нули због тога што услед јединичног оптерећења у штаповима нема сила

$X_1^p (= \bar{v}_0) = 0$

$X_2^p = \bar{v}_{(2)} = -\sum \bar{C}_i c_i =$

$= -(-1) \cdot 0{,}03 = 0{,}03$

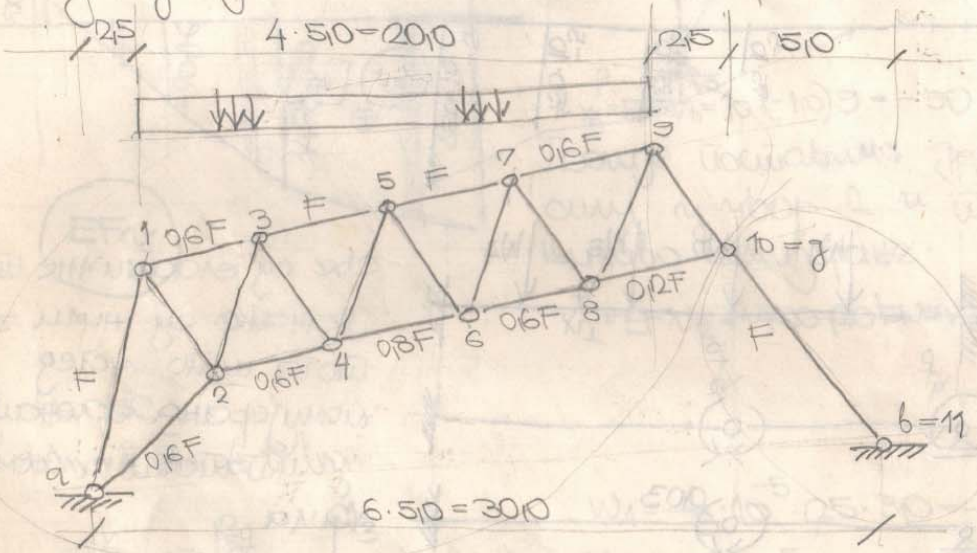


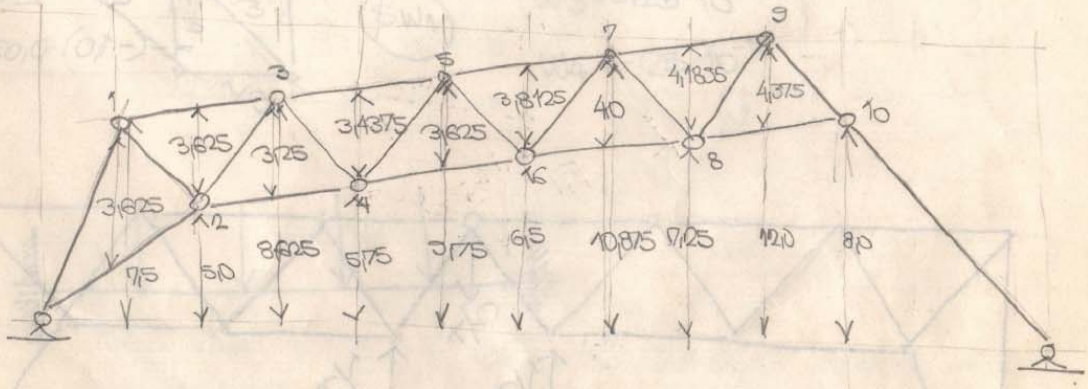
$$\Delta l_{ab} = 0$$

$$\Delta l_{ab} = \sum_{m=1}^{n} \Delta l_m \cdot \sec\alpha_m + \sum_{m=1}^{n} w_m \cdot y_m = 0$$

$$w_g = -\frac{1}{y_g}\left(\sum_{m=1}^{n} \Delta l_m \cdot \sec\alpha_m + \sum_{m=j}^{n} w_m y_m\right)$$

~~ЗАДАТАК~~ За решеткасти носач на слици нацртати дијаграм помјерања доњег појаса услед датог спољашњег оптерећења



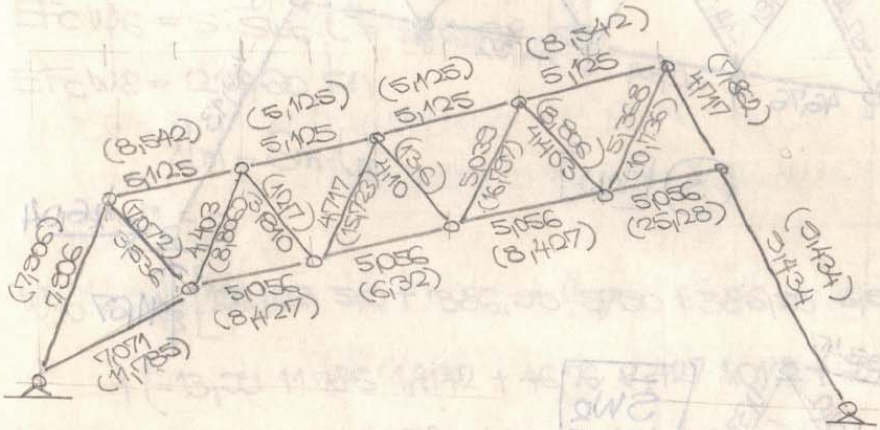


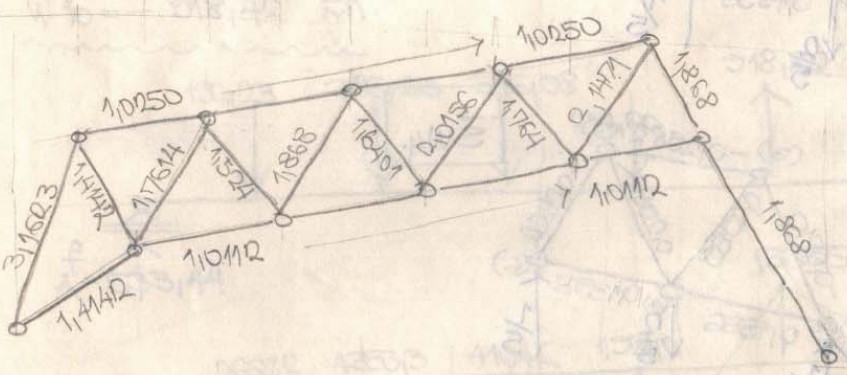
Regulowane zyderki

l_w , l_w'

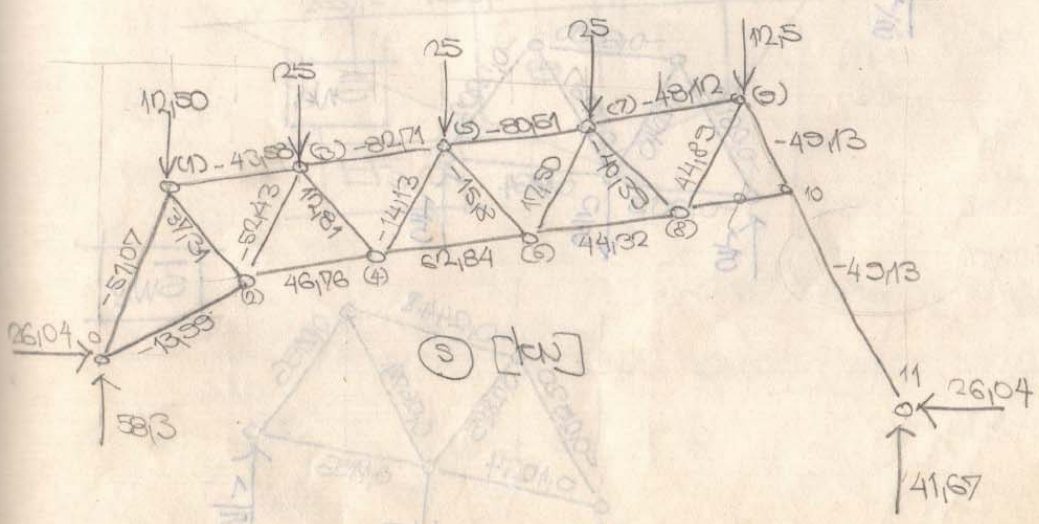
$l_w' = \frac{F_c}{F_m} \cdot l_w \qquad F_c = F$

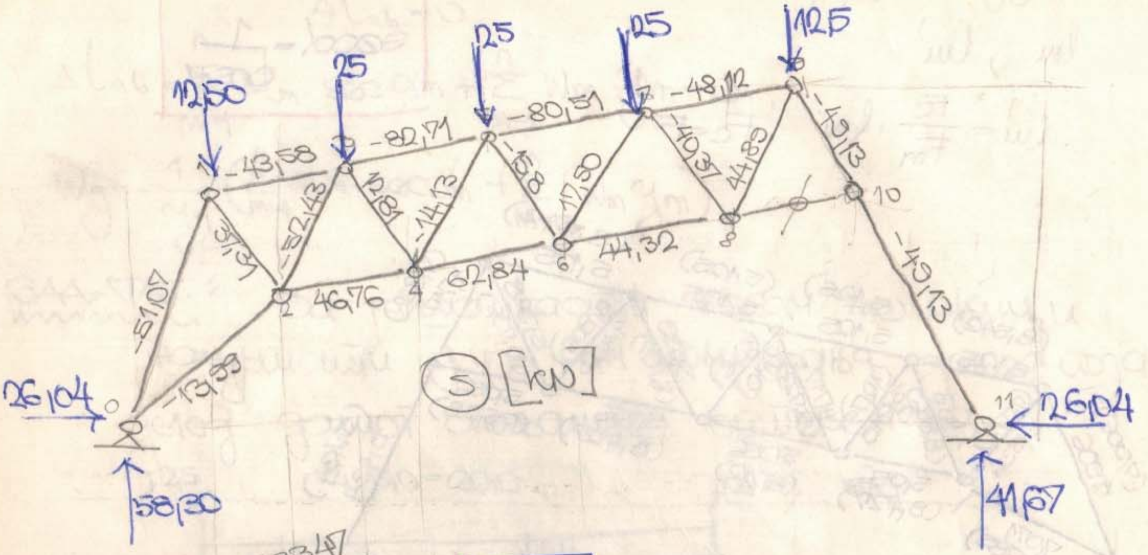
$\sec\alpha = \frac{1}{\cos\alpha}$

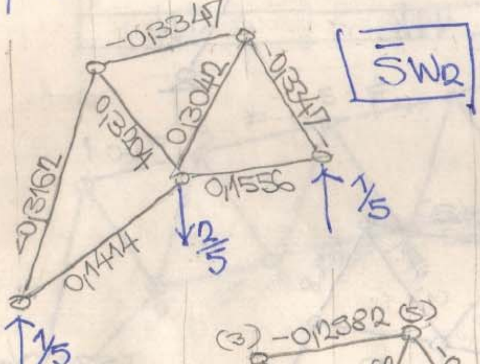


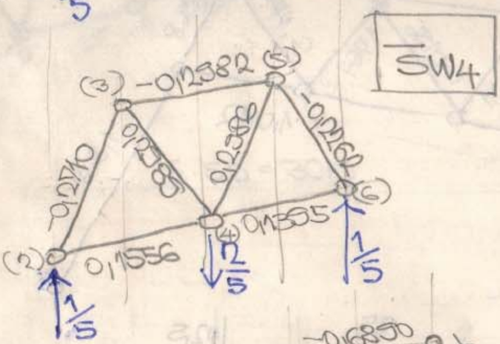


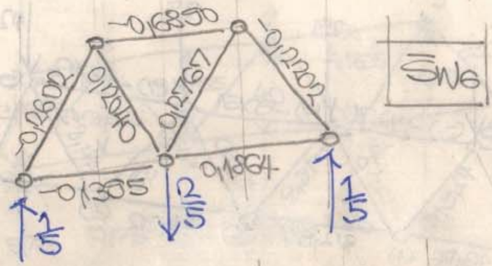
sec dm

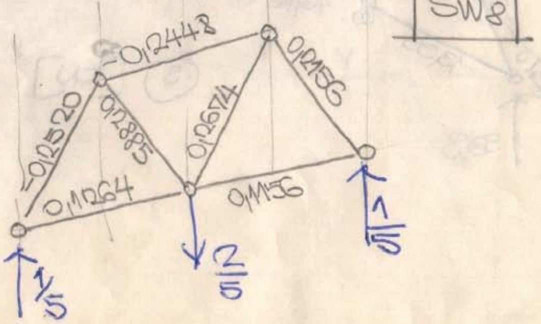



12,50
25
25
25
12,5
-43,58
-82,71
-80,51
-48,12
-51,07
37,31
-52,43
15,81
-14,13
-15,8
17,80
-40,37
44,83
-49,13
-49,13
-13,33
46,76
62,84
44,32
26,104
26,104
58,30
41,67
S [kN]


-0,3347
-0,3347
0,3004
0,3042
-0,3162
0,1556
0,1414
1/5
2/5
1/5
S̄w2


(3) -0,2582 (5)
-0,2710
0,2385
0,2386
-0,2160
(2)
(4)
(6)
0,1556
0,1355
1/5
2/5
1/5
S̄w4


-0,6850
-0,2602
0,2040
0,2767
-0,2202
-0,1355
0,1864
1/5
2/5
1/5
S̄w6


-0,2448
-0,2520
0,2885
0,2674
0,2156
0,1264
0,1156
1/5
2/5
1/5
S̄w8

$$W_m = \sum_{m=1}^{n} \frac{S_m \cdot \bar{S}_m}{EF} \cdot l_m$$

$EF_c \cdot W_2 = 136{,}23$ tm

$EF_c W_4 = 386{,}35$ tm

$EF_c W_6 = S \cdot \bar{S}_{W_6} \cdot l = 386{,}38$ tm

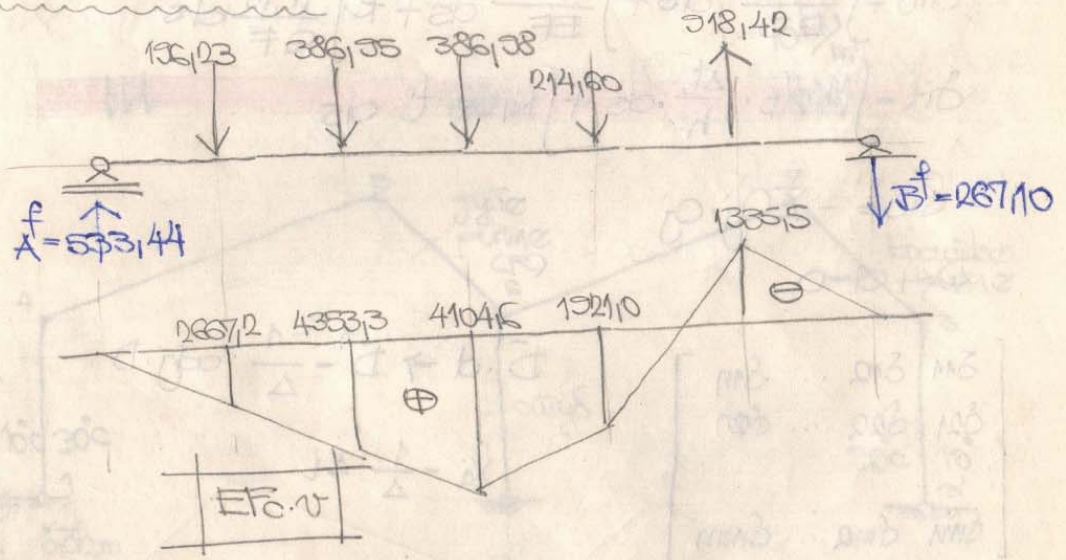
$EF_c W_8 = 214{,}60$ tm

$\Delta l_m = S_m \cdot l_m$

$\Delta l_{ab} = 0$

$$W_{10} = -\frac{1}{8}\Big[136{,}23 \cdot 5{,}10 + 386{,}35 \cdot 5{,}750 + 386{,}38 \cdot 6{,}50 + 214{,}6 \cdot 7{,}125 +$$
$$+ (-13{,}30 \cdot 11{,}785 \cdot 1{,}4142 + 46{,}76 \cdot 8{,}427 \cdot 1{,}0112 + 62{,}84 \cdot 6{,}32 \cdot 1{,}012 +$$
$$+ 44{,}32 \cdot 8{,}427 \cdot 1{,}0112 - 49{,}13 \cdot 9{,}434 \cdot 1{,}8868)\Big] =$$

$W_{10} = -318{,}42$ tm



ЗАДАТАК:

ЗА НОСАЧ ПРЕМА СКИЦИ УСЛЕД ЗАГРИЈАВАЊА ГОРЊЕГ ВЛАКНА ЗА $t_0 = 20°C$

1. НАЦРТАТИ ДИЈАГРАМ ВЕРТИКАЛНИХ ПОМЕРАЊА СА ОРДИНАТАМА У ЧЕСТИНАМА РАСПОНА И ОДРЕДИТИ ХОРИЗОНТАЛНО ПОМЕРАЊЕ ОСЛОНЦА Б.

НОСАЧ ЈЕ ПРАВОУГАОНОГ ПОПРЕЧНОГ ПРЕСЈЕКА И ТО: СИО 0–2 $b/h = 0{,}4/1{,}0$ m

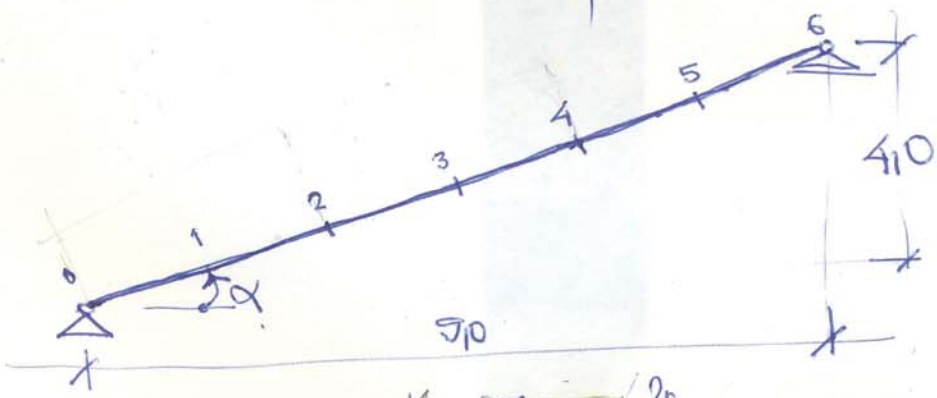
2–4 $b/h = 0{,}4/0{,}8$ m

4–6 $b/h = 0{,}4/0{,}6$ m

НАПОМЕНА:

ТЕМПЕРАТУРА СЕ ПО ВИСИНИ ПОПРЕЧНОГ ПРЕСЈЕКА МИЈЕЊА ЛИНЕАРНО ОД $t_0 = 20°C$ НА ГОРЊЕМ ВЛАКНУ ДО $t_u = 0°C$ НА ДОЊЕМ ВЛАКНУ

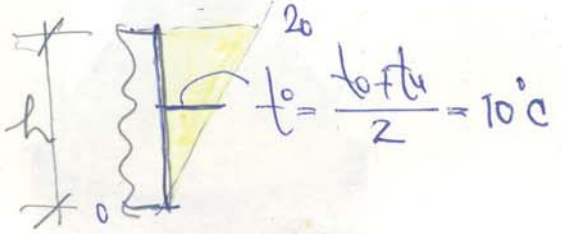
$\alpha_t = 10^{-5} \frac{1}{°C}$ $E = 3 \cdot 10^7 kN/m^2$ · $t_0 = 20°C$



$\cos\alpha = 0{,}9138$

$\sin\alpha = 0{,}4061$

$tg\alpha = 0{,}444$



$t° = \frac{t_0 + t_u}{2} = 10°C$

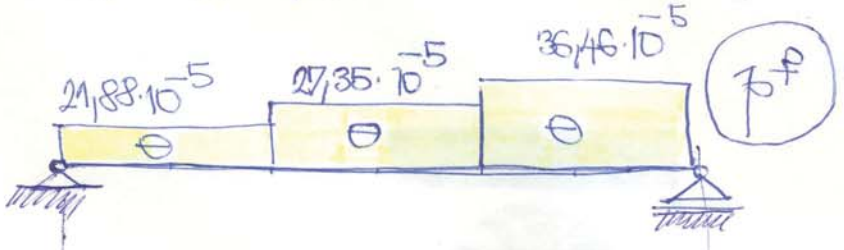
$\Delta t = 0 - 20 = -20°C$

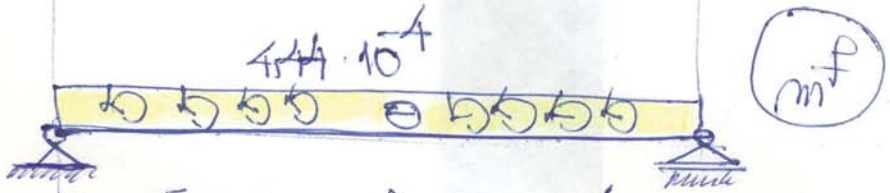
$\beta^f = \frac{\alpha_t \cdot \Delta t}{h \cdot \cos\alpha}$

$m^f = \alpha_t \cdot t° \cdot tg\alpha$

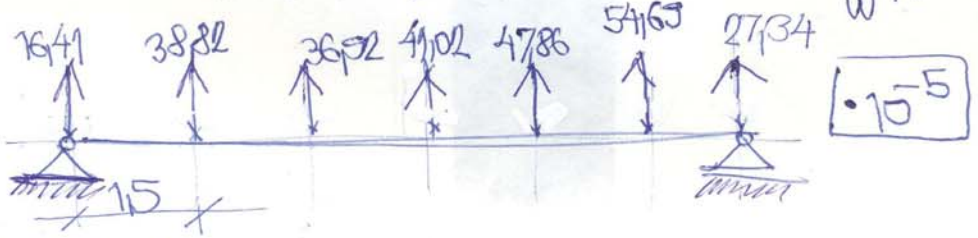
ФИКТИВНИ НОСАЧ

$$\frac{\alpha_t \cdot \Delta t}{h \cdot \cos\delta} = \frac{10^{-5}(-20)}{h_0 \cdot 0,9738} = -21,88 \cdot 10^{-5}$$





$$m^f = 10^{-5} \cdot 10 \left(-0,444\right) = -44 \cdot 10^{-4}$$



$$w_1^{mf} = \frac{4,44 + 4,44}{2} - \frac{4,44 + 4,44}{2} = 0$$