9

СТАТИКА КОНСТРУКЦИЈА

Модул: Хидротехника и водно инжењерство околине, Саобраћајнице, Архитектонско инжењерство

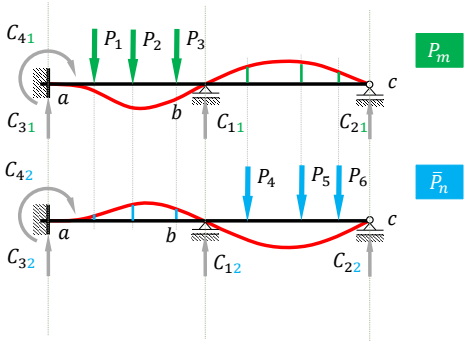
– материјал за вежбе –

2024.

Теореме о узајамности

20 Betti – јева теорема

$$\sum P_m \bar{\delta}_m + \sum C_i \bar{c}_i = \sum \bar{P}_n \delta_n + \sum \bar{C}_i c_i$$



Рад спољашњих сила P_m и C_i **првог** система при померањима која изазива **други** систем једнак је раду спољашњих сила $\bar{P}_n$ и $\bar{C}_i$ **другог** система при померањима који изазива **први** систем утицаја.

21 Maxwell – ова теорема

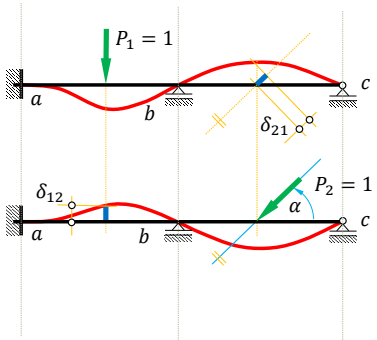
$$\sum P_m \bar{\delta}_m + \sum C_i \bar{c}_i = \sum \bar{P}_n \delta_n + \sum \bar{C}_i c_i$$

$P_m \Rightarrow P_1 = 1 \qquad \Rightarrow \quad \delta_n = \delta_{21}$

$\bar{P}_n \Rightarrow P_2 = 1 \qquad \Rightarrow \quad \bar{\delta}_m = \delta_{12}$

$$\sum 1 \cdot \bar{\delta}_{12} + \sum C_i \bar{c}_i = \sum 1 \cdot \delta_{21} + \sum \bar{C}_i c_i$$

$$1 \cdot \delta_{12} = 1 \cdot \delta_{21} \qquad \delta_{12} = \delta_{21}$$



Померање нападне тачке силе P_1 у правцу те силе услед силе P_2 једнако је померању нападне тачке силе P_2 у правцу силе P_2 услед силе P_1.

22 прва Rayleigh – јева теорема

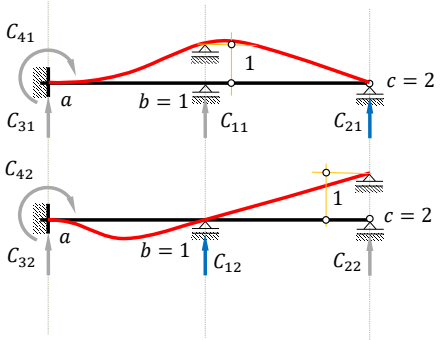
$$\sum P_m \bar{\delta}_m + \sum C_i \bar{c}_i = \sum \bar{P}_n \delta_n + \sum \bar{C}_i c_i$$

$P_m \Rightarrow P_1 = 0 \qquad c_i = c_1 = 1 \quad \Rightarrow \quad C_i = C_{12}$

$\bar{P}_n \Rightarrow P_2 = 0 \qquad \bar{c}_i = c_2 = 1 \quad \Rightarrow \quad \bar{C}_i = C_{21}$

$$\sum 0 \cdot \bar{\delta}_m + \sum C_i \bar{c}_i = \sum 0 \cdot \delta_n + \sum \bar{C}_i c_i$$

$$C_{12} \cdot 1 = C_{21} \cdot 1$$



Реакција ослонца 1 услед јединичног померања ослонца 2 једнака је реакцији ослонца 2 услед јединичног померања ослонца 1.

23 друга Rayleigh – јева теорема

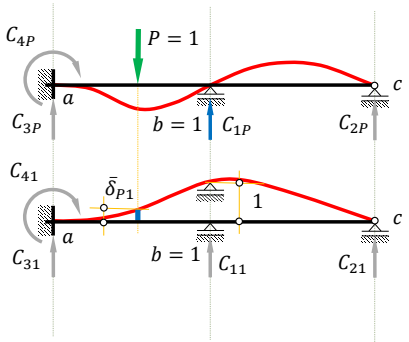
$$\sum P_m \bar{\delta}_m + \sum C_i \bar{c}_i = \sum \bar{P}_n \delta_n + \sum \bar{C}_i c_i$$

$P_m \Rightarrow P = 1 \qquad c_i = 0 \quad \Rightarrow \quad C_{1P}$

$\bar{P}_n \Rightarrow P = 0 \qquad \bar{c}_i = c_1 = 1 \quad \Rightarrow \quad \bar{\delta}_{P1}$

$$\sum 1 \cdot \bar{\delta}_{P1} + \sum C_{1P} \cdot 1 = \sum 0 \cdot \delta_n + \sum \bar{C}_{1P} \cdot 0$$

$$C_{1P} \cdot 1 = -\bar{\delta}_{P1} \cdot 1 \qquad C_{1P} = -\bar{\delta}_{P1}$$



Реакција ослонца **1** услед дејства јединичне силе **P** једнака је негативној вредности померања нападне тачке силе **P** у правцу те силе услед јединичног померања ослонца **1**.

DEFORMACIJA STATIČKI ODREĐENIH NOSAČA

$$\left.\begin{aligned} \varkappa &= \frac{M}{EJ} + \alpha_t \frac{\Delta t}{h} \\ \varepsilon &= \frac{N}{EF} + \alpha_t t^\circ \\ \varphi_T &= k \frac{T}{GF} \end{aligned}\right\} \rightarrow \Delta_{ik}, \tau_{ik}, \tau_{ki} \rightarrow C_{oi}, C_{ui} \Rightarrow u_i, v_i$$

- princip virtuelni radova =

Princip: Algebarski zbir radova unutrašnji i spoljašnji jednog mogućeg stanja jeste jednak nuli

$$\underbrace{\Sigma \bar{F}_i + \Sigma \bar{C}_i c_i}_{\delta} = \int_s (\bar{M}\varkappa + \bar{N}\varepsilon + \bar{T}\varphi_T)\, ds$$

$\bar{P}=1$
$\bar{M}=1$

$$\delta = \int_s (\bar{M}\varkappa + \bar{N}\varepsilon + \bar{T}\varphi_T)\, ds - \Sigma \bar{C}_i c_i$$

$$\delta = \int_s \left(\frac{M\bar{M}}{EJ} + \frac{N\bar{N}}{EF} + k\frac{T\bar{T}}{GF} + \bar{M}\alpha_t \frac{\Delta t}{h} + \bar{N}\alpha_t t^\circ \right) ds - \Sigma \bar{C}_i c_i$$

$$\delta_0 = \int_s \frac{M\bar{M}}{EJ} ds + \int_s \frac{N\bar{N}}{EF} ds + k \cdot \int_s \frac{T\bar{T}}{GF} ds \;/ EJ_c$$

$$\delta_t = \int_s \bar{M}\alpha_t \frac{\Delta t}{h} ds + \int_s \bar{N}\alpha_t t^\circ ds$$

$\delta_c = -\Sigma \bar{C}_i c_i$ / pom. oslonca

$$EJ_c \cdot \delta_0 - \int_s M\bar{M}\frac{J_c}{J} ds + \frac{J_c}{F_c}\int_s N\bar{N}\frac{F_c}{F} ds + (2+2\mu)\frac{J_c}{F_c}\int_s T\bar{T}\frac{F_c}{F} ds$$

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$$ds' = \frac{J_c}{J} ds$$

$$ds'' = \frac{F_c}{F} ds$$

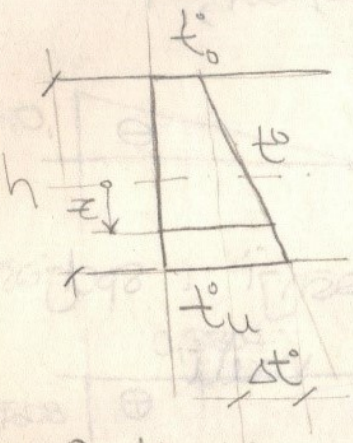
$$ds''' =$$

$$\boxed{EJ_c \delta_0 - \int_s M\bar{M}\frac{J_c}{J} ds + \frac{J_c}{F_c}\int_s N\bar{N}\frac{F_c}{F} ds}$$

usled spoljašnjeg opterećenja

$$EJ_c \cdot \delta_t = EJ_c \int_s \bar{M} \alpha_t \frac{\Delta t}{h} ds + EJ_c \int_s \bar{N} \alpha_t t^\circ \cdot ds$$

$$EJ_c \delta_c = -\Sigma C_i c_i$$



$$t^\circ(z) = t^\circ_0 + \Delta t \frac{z}{h}$$

$$\Delta t^\circ = t^\circ_u - t^\circ_0$$

$$t^\circ = \frac{t^\circ_u + t^\circ_0}{2}$$

tod rēšetki 5

$$\delta = \int_s \bar{N} \varepsilon ds - \Sigma \bar{C}_i c_i$$

$$\delta = \sum_s \int_i^t \bar{N} \varepsilon ds - \Sigma C_i c_i = \sum_s \bar{S} \Delta l - \Sigma \bar{C}_i c_i = \Sigma \frac{s\bar{s}}{EF} l + \Sigma \bar{S} \cdot \alpha_t t^\circ \cdot l - \Sigma \bar{C}_i c_i$$

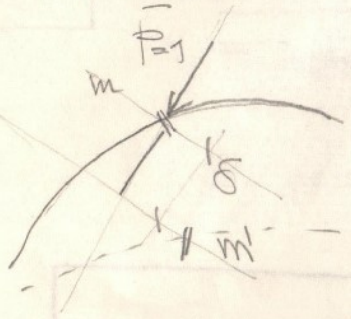
$$\int_i^t \bar{N} \varepsilon ds = \bar{N} \varepsilon \int_i^t ds = \bar{N} \cdot \underbrace{\varepsilon \cdot l}_{\Delta l} = \bar{s} \cdot \Delta l$$

$$\delta_0 = \sum_s \frac{s\bar{s}}{EF} l \Big/ EF_c \rightarrow EF_c \cdot \delta_0 = \Sigma \frac{s\bar{s}}{F} \cdot l$$

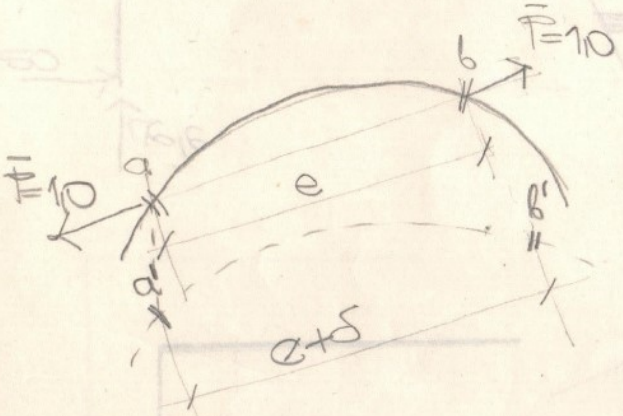
$$\delta_t = \sum_s \bar{S} \alpha_t t^\circ l \Big/ EF_c \rightarrow EF_c \cdot \delta_t = \Sigma \bar{S} \alpha_t \cdot t^\circ l EF_c$$

$$\delta_c = -\Sigma \bar{C}_i c_i \Big/ EF_c \qquad EF_c \delta_c = -EF_c \Sigma \bar{C}_i c_i$$

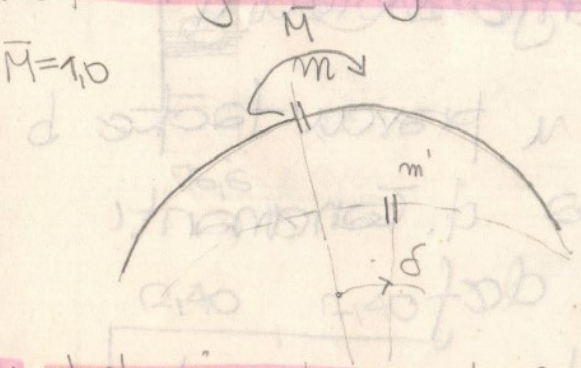
1. Jed. pom. tačke u određenom pravcu



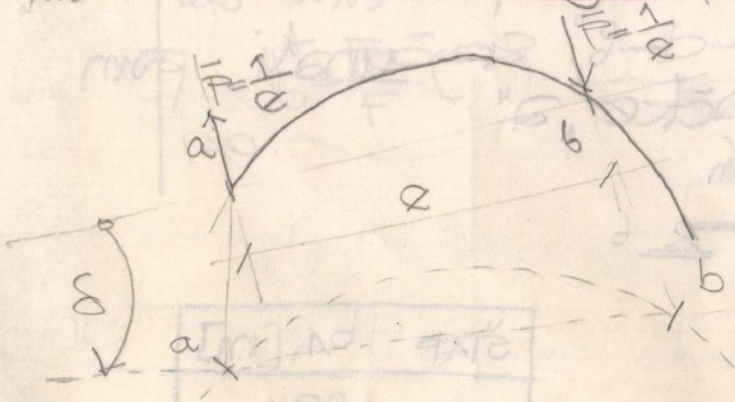
2. Promjena odstojanja između dvije tačke



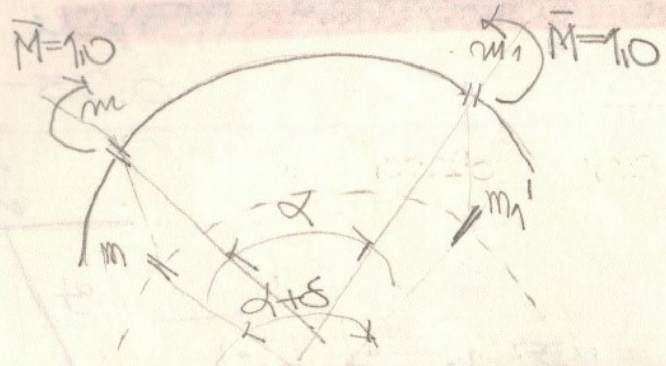
3. Promjena ugla u presjeku



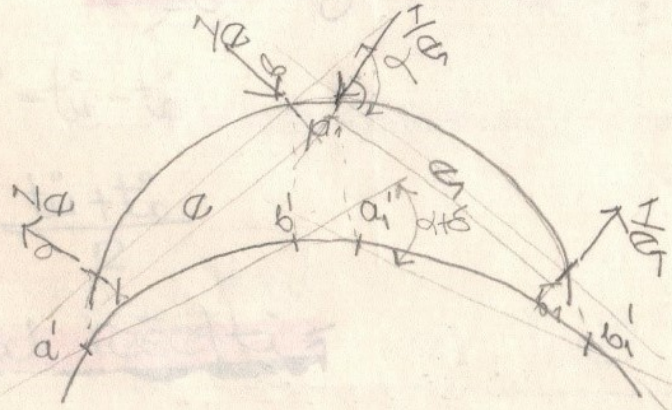
4. Obrtanje prave koja prolazi kroz tačke A i B



5. promjena ugla između tangenti

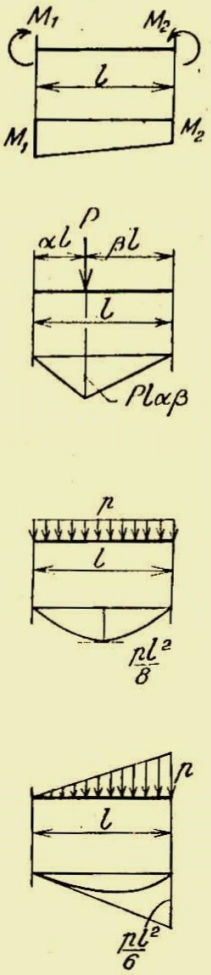


6. promjena ugla između dvije prave



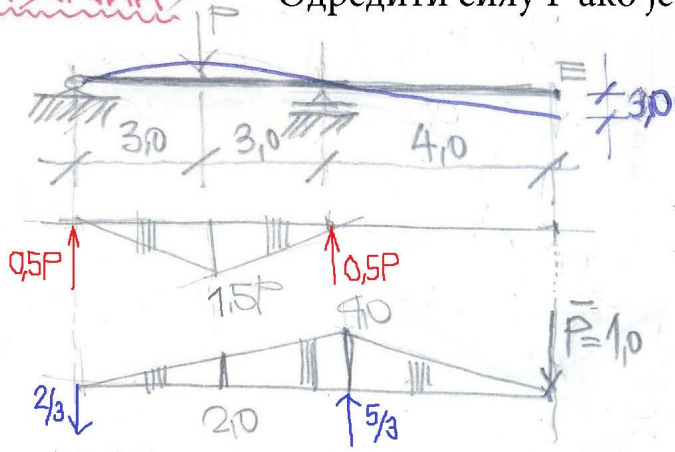
TABLICA 1

$\frac{1}{EJ}\int M\bar{M}ds$	$\bar{i}$	$\bar{k}$	$\bar{i}$ $\bar{k}$	αl βl		$\bar{k}$
i	$i\bar{i}$	$\frac{1}{2}i\bar{k}$	$\frac{1}{2}i(\bar{i}+\bar{k})$	$\frac{1}{2}i\bar{m}$	$\frac{2}{3}i\bar{m}$	$\frac{1}{4}i\bar{k}$
k	$\frac{1}{2}k\bar{i}$	$\frac{1}{3}k\bar{k}$	$\frac{1}{6}k(\bar{i}+2\bar{k})$	$\frac{1}{6}k\bar{m}(1+\alpha)$	$\frac{1}{3}k\bar{m}$	$\frac{2}{15}k\bar{k}$
i	$\frac{1}{2}i\bar{i}$	$\frac{1}{6}i\bar{k}$	$\frac{1}{6}i(2\bar{i}+\bar{k})$	$\frac{1}{6}i\bar{m}(1+\beta)$	$\frac{1}{3}i\bar{m}$	$\frac{7}{60}i\bar{k}$
i k	$\frac{1}{2}(i+k)\bar{i}$	$\frac{1}{6}(i+2k)\bar{k}$	$\frac{1}{6}[i(2\bar{i}+\bar{k})+$ $+k(\bar{i}+2\bar{k})]$	$\frac{1}{6}[i(1+\beta)+$ $+k(1+\alpha)]\bar{m}$	$\frac{1}{3}(i+k)\bar{m}$	$\frac{1}{60}(7i+8k)\bar{k}$
$\bar{m}$ αl βl	$\frac{1}{2}m\bar{i}$	$\frac{1}{6}m\bar{k}(1+\alpha)$	$\frac{1}{6}m[\bar{i}(1+\beta)+$ $+\bar{k}(1+\alpha)]$	$\frac{1}{3}m\bar{m}$	$\frac{1}{3}m\bar{m}(1+\alpha\beta)$	$\frac{1}{20}m\bar{k}(1+\alpha)\times$ $\times(\frac{7}{3}-\alpha^2)$
$\bar{m}$	$\frac{2}{3}m\bar{i}$	$\frac{1}{3}m\bar{k}$	$\frac{1}{3}m(\bar{i}+\bar{k})$	$\frac{1}{3}m\bar{m}(1+\alpha\beta)$	$\frac{8}{15}m\bar{m}$	$\frac{1}{5}m\bar{k}$
k	$\frac{1}{4}k\bar{i}$	$\frac{2}{15}k\bar{k}$	$\frac{1}{60}k(7\bar{i}+8\bar{k})$	$\frac{1}{20}k\bar{m}(1+\alpha)\times$ $\times(\frac{7}{3}-\alpha^2)$	$\frac{1}{5}k\bar{m}$	$\frac{8}{105}k\bar{k}$
i	$\frac{1}{4}i\bar{i}$	$\frac{7}{60}i\bar{k}$	$\frac{1}{60}i(8\bar{i}+7\bar{k})$	$\frac{1}{20}i\bar{m}(1+\beta)\times$ $\times(\frac{7}{3}-\beta^2)$	$\frac{1}{5}i\bar{m}$	$\frac{31}{420}i\bar{k}$



ЗАДАТАК:

Одредити силу P ако је вертикално померање тачке "E" 3mm.



$v_E = 3{,}0\ mm$

$EJ = 30 \cdot 10^3\ kNm^2$

$M\ (kNm)$

$M_1\ (kNm)$

$EJ v_E = -9{,}0P$

$30 \cdot 10^3 \cdot 0{,}003 = -9{,}0P$

$P = -10{,}00\ kN$

$EJ v_E = -\frac{1}{3} \cdot 1{,}5P \cdot 2 \cdot 3 - \frac{1}{6} 1{,}5P(4+2\cdot 2) \cdot 3 =$

$= -3P - 6P = -9P$

zadatak:

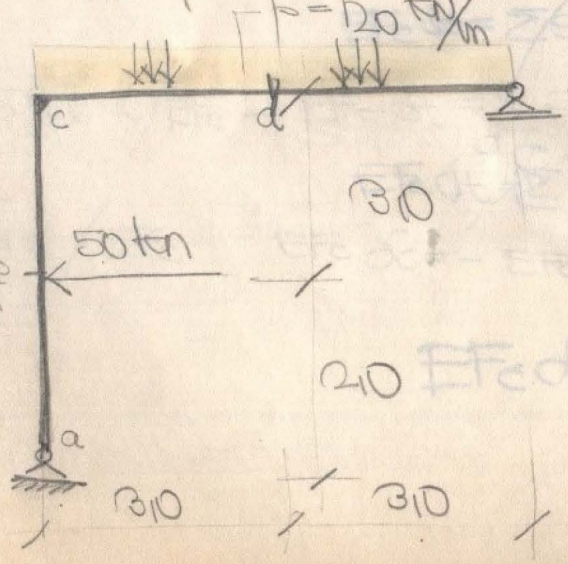
Za dati nosač i opterećenje odrediti"

1.

a) pomeranje tačke f u pravcu tačke b

b) obrtanje preseka d. Tenemariti uticaj transf. sila na def.

2. Pri zadatoj promjeni temperature $t_b = -5°C$, $t_u = 25°C$ duž štapa „c–d–b" sračunati pom. tačke „d" u pravcu tačke „2"



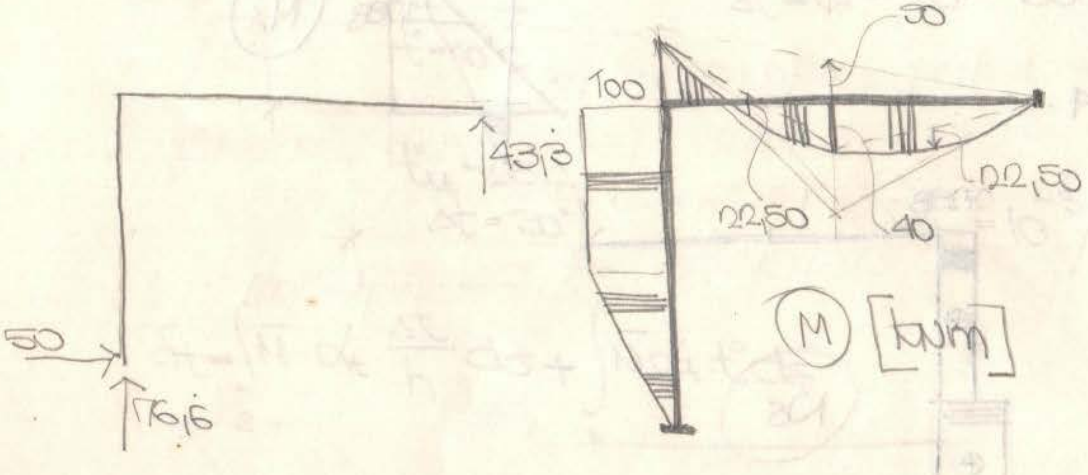
ŠTAP	b/h [m]	J (m⁴)
a–c	0,3/0,4	0,0016
c–b	0,3/0,5	0,003125

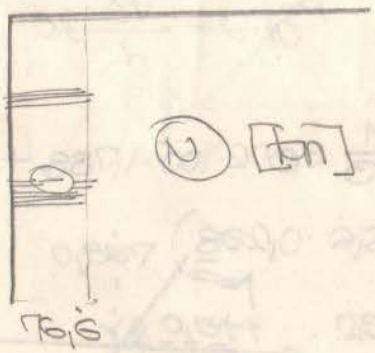
$E = 2 \cdot 10^7\ kN/m^2$

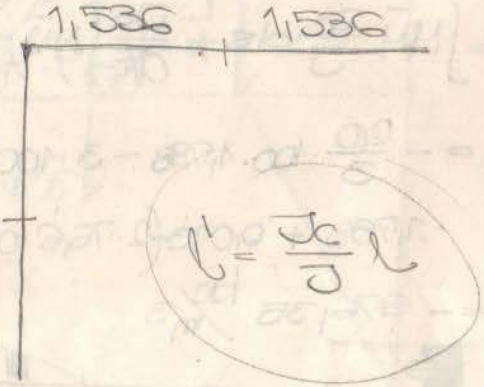
F (m²)
0,12
0,15

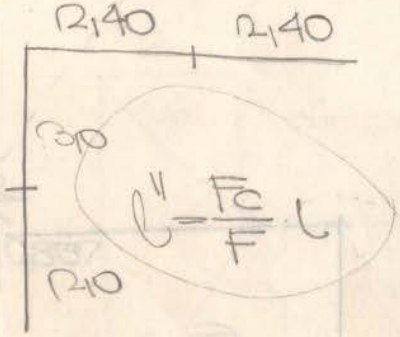
$F_c = 0{,}12\ m^2$ $\quad J_c = 0{,}0016\ m^4$ $\quad \frac{J_c}{F_c} = 0{,}0133\ m^2$

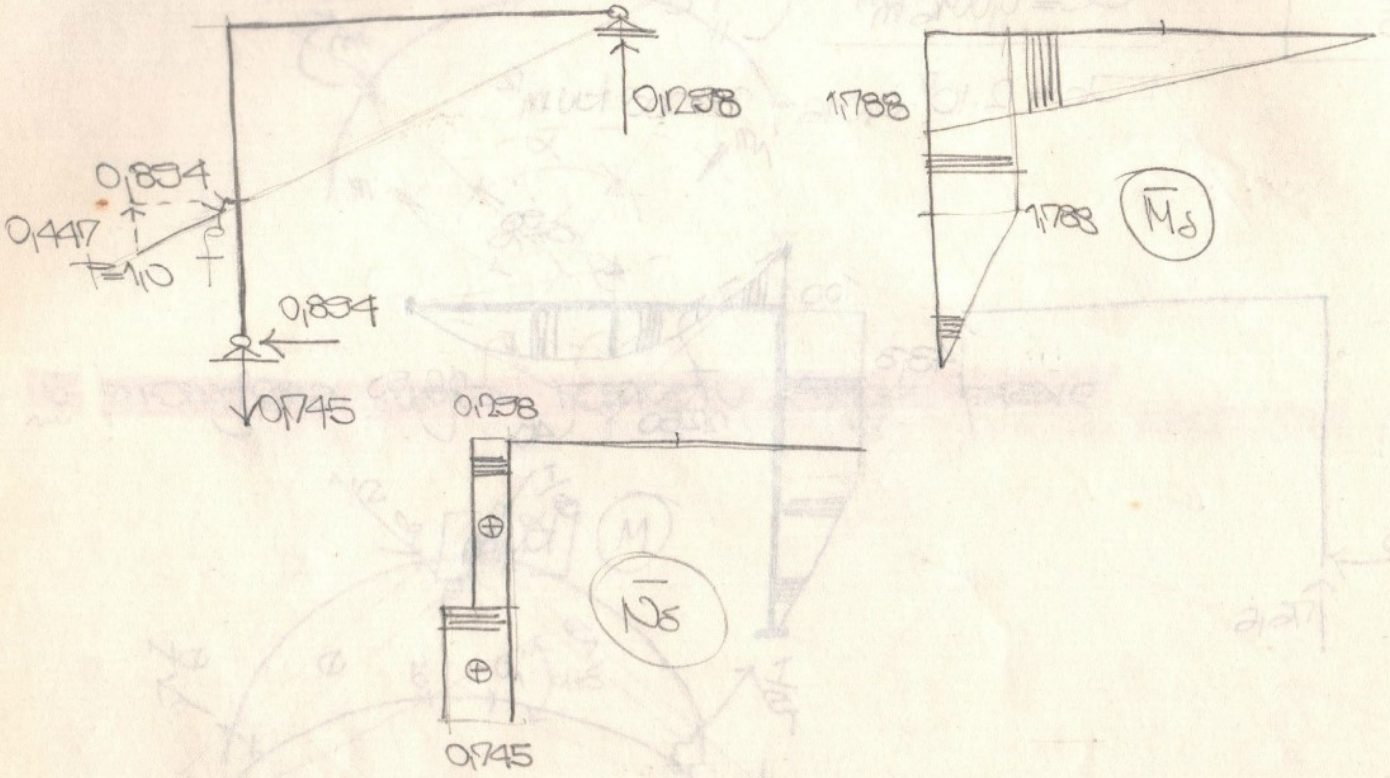
$EJ_c = 2 \cdot 10^7 \cdot 0{,}0016 = 32000\ kNm^2$











$$EJ_c\delta = \int M\bar{M}\frac{J_c}{J}ds + \frac{J_c}{F_c}\int N\bar{N}\frac{F_c}{F}ds$$

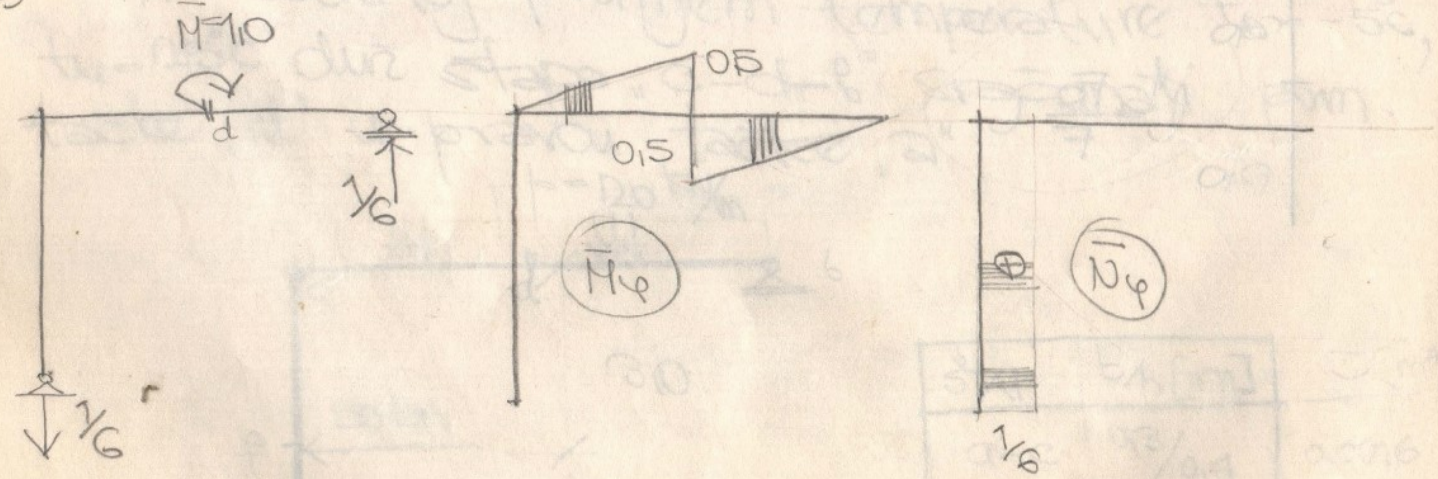
$$EJ_c\cdot\delta = -\frac{2,0}{3}\cdot 100\cdot 1,788 - 3\cdot 100\cdot 1,788 - \frac{1}{3}3,072\cdot 100\cdot 1,788 + \frac{1}{3}3,072\cdot 30\cdot$$

$$\cdot 1,788 + 0,013(2\cdot 76,6\cdot 0,1745 - 3\cdot 76,6\cdot 0,1298)$$

$$EJ_c\cdot\delta = -676,135\ \frac{tm}{m^3}$$

$$\delta = -0,021\,m = -2,1\,cm$$

b)

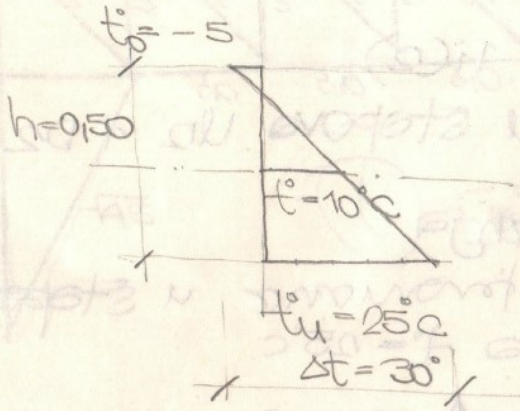


$$EJ_c\varphi = -\frac{1}{6}1,536\cdot 0,50(2\cdot 40 - 100) - \frac{1}{3}1,536\cdot 0,50\cdot 22,50 + \frac{1}{3}1,536\cdot 40\cdot 0,50 +$$

$$+\frac{1}{3}1,536\cdot 22,50\cdot 0,50 + 0,013\left(-50\cdot 76,6\cdot\frac{1}{6}\right) = 101,55\ tm\,m2$$

$\varphi = 0{,}0032$ rad.

12.

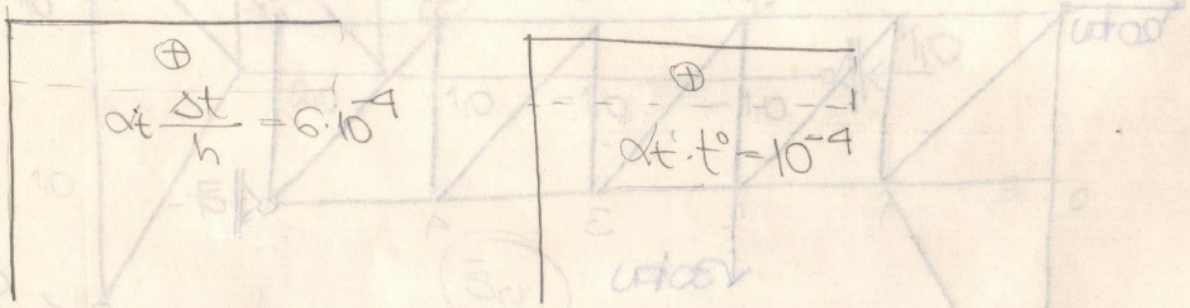


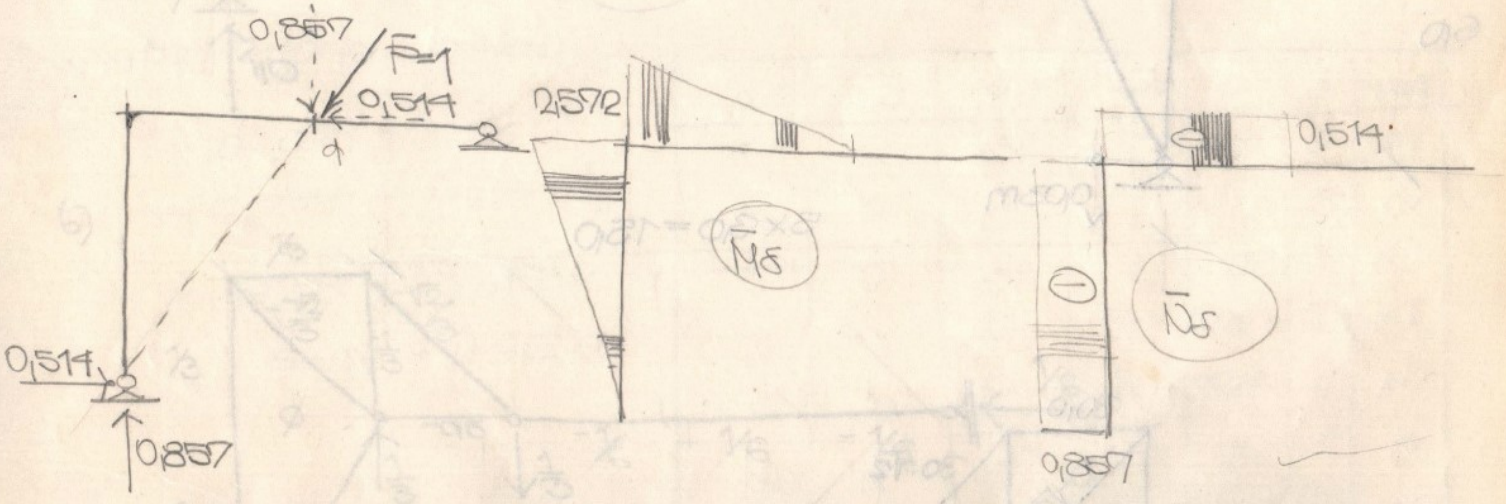
$$t° = \frac{t_u + t_b}{2} = 10°C$$

$$\Delta t = t_u - t_b = 30°C$$

$\alpha_t = 10^{-5}/°C$ - za beton

$$\delta_t = \int_s \bar{M} \cdot \alpha_t \frac{\Delta t}{h} ds + \int_s \bar{N} \alpha_t \cdot t° ds$$





$$EJ_c \delta_t = \left(-\frac{1}{2} \cdot 3{,}0 \cdot 2572 \cdot 6 \cdot 10^{-4} - 3 \cdot 0514 \cdot 10^{-4}\right) \cdot EJ_c$$

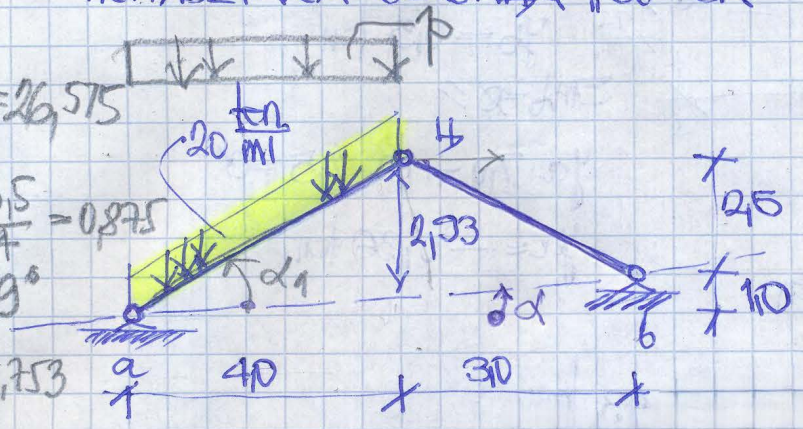
$$\delta_t = -2{,}161 \cdot 10^{-3} \text{ m} \rightarrow \delta = -2{,}16 \text{ mm}$$

ЗАДАЧА:

ЗА НОСАЧ НА СЛИКИ УСЛЕД ЗАДАТОГ ОПТЕРЕЋЕЊА ОДРЕДИТИ:

1° – ПРОМЕНУ УГЛА У „Ц"

2° – ХОРИЗОНТАЛНО ПОМЕРАЊЕ ЧВОРА „Ц" УСЛЕД ВЕРТИКАЛНОГ ПОМЕРАЊА ОСЛОНЦА „a" ЗА 0,01 m



$p = \frac{20}{\cos\alpha_1} = 26,575$

$\operatorname{tg}\alpha_1 = \frac{3,5}{4} = 0,875$

$\alpha_1 = 41,19°$

$\cos\alpha_1 = 0,753$

$E = 2 \cdot 10^7 \text{ kNm}^2$

$J = 0,00072 \text{ m}^4$

$\operatorname{tg}\alpha = \frac{1}{7} = 0,14285 = 0,14$

$\alpha = 8,13°$

$\Sigma M_a = 0 \Rightarrow y_b \cdot 7,10 - 106,30 \cdot 2,10 = 0 \rightarrow y_b = 30,37$ tcn

$\Sigma M_b = 0 \Rightarrow -y_a \cdot 7,10 + 106,30 \cdot 5,10 = 0 \Rightarrow y_a = 75,93$ tcn

$H_a = \frac{1}{2,93}(75,93 \cdot 4,10 - 106,30 \cdot 2,10) \Rightarrow H_a = 31,10$ tcn

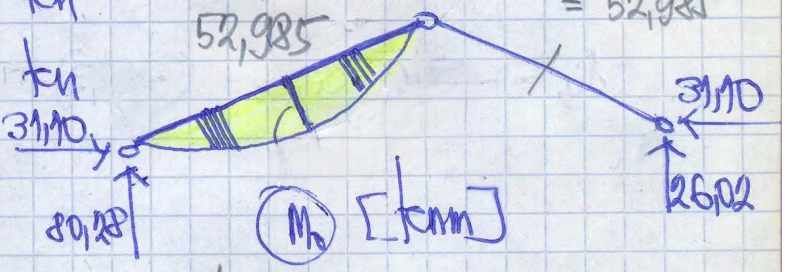
$H_b = \frac{1}{2,93}(30,37 \cdot 3,10) \Rightarrow H_b = 31,10$ tcn

$V_a = 75,93 + 31,10 \cdot 0,14 = 80,28$ tcn

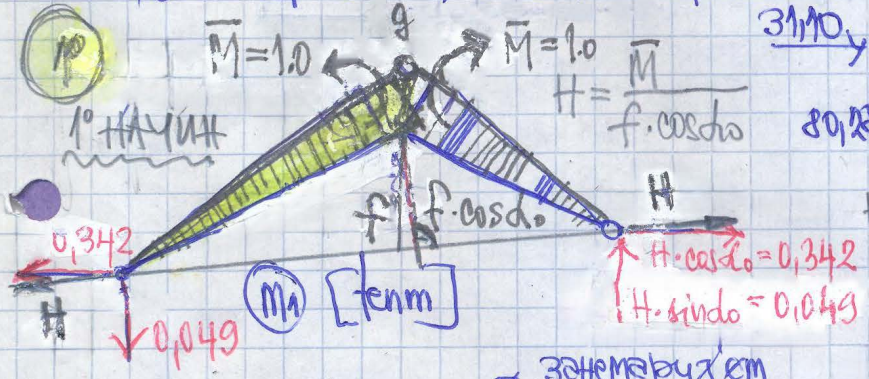
$V_b = 30,37 - 31,10 \cdot 0,14 = 26,02$ tcn

(p) $M = 80,28 \cdot 2 - 31,1 \cdot 0,5 \cdot 3,5 - 26,575 \cdot 2 \cdot 1 =$

$= 52,985$



1° НАЧИН



$H = \frac{\bar{M}}{f \cdot \cos\alpha}$

$H = \frac{1}{2,93 \cdot 0,9899} = 0,345$ tcn

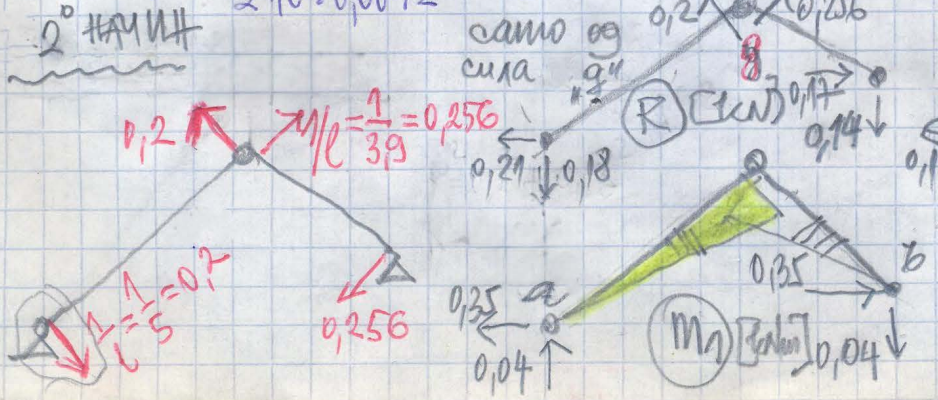
$EI \cdot \delta_y = \int_S \frac{M_1 M_0}{EJ} ds + \int_S \frac{N_1 N_0}{EF} ds + \int_S \frac{T_1 T_0}{GF} ds$ (занемарујемо)

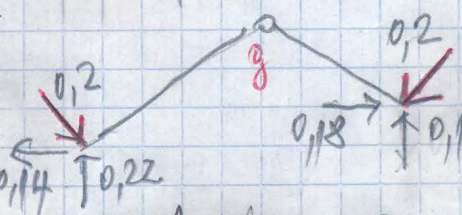
$EI \cdot \delta_y = -\frac{1}{3} \cdot 1 \cdot 52,985 \cdot 5 + 0 = 88,417$

$\delta_y = \frac{88,417}{2 \cdot 10^7 \cdot 0,0072}$

$\delta_y = 0,000614$ rad. → ПРОВЕРА СА 2° НАЧИНОМ.

2° НАЧИН

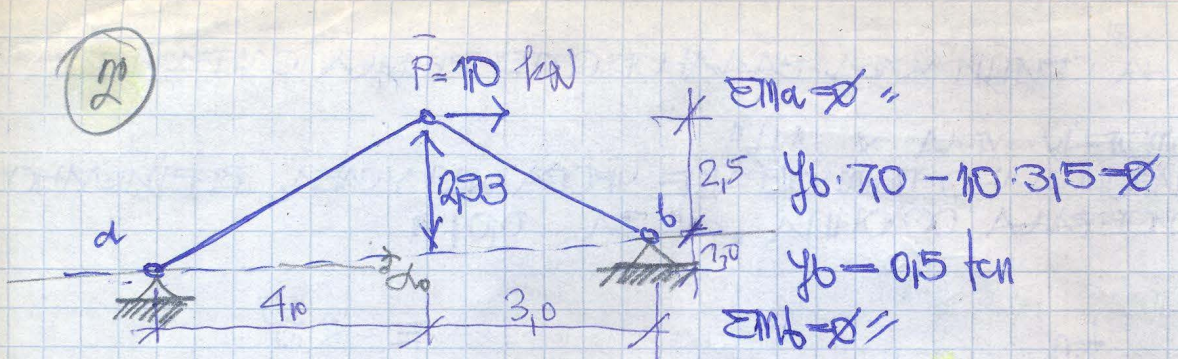




$EI\delta_y = \frac{1}{3} \cdot 1 \cdot 52,985 \cdot 5 + 0 = 88,417$

$\delta_y = 0,000614$ rad.

2°



$\Sigma M_a = 0$ =

$y_b \cdot 7,0 - 1,0 \cdot 3,5 = 0$

$y_b = 0,5$ kN

$\Sigma M_b = 0$ =

$y_a \cdot 7,0 + 1,0 \cdot 2,5 = 0$

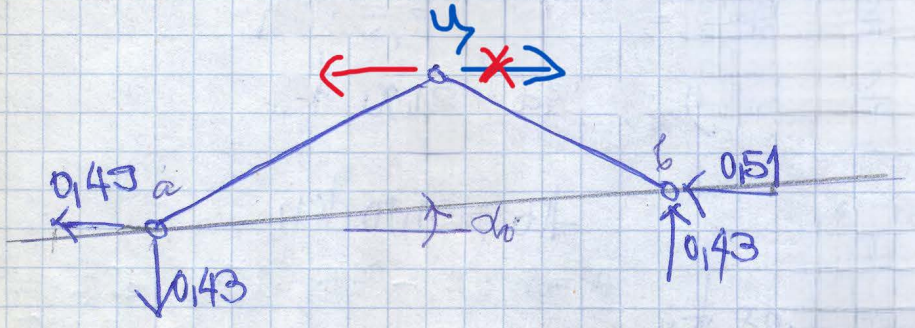
$y_a = -0,36$ kN

$H_b = \frac{1}{2,93}(0,5 \cdot 3,0) = 0,51$ kN

$H_a = \frac{1}{2,93}(-0,36 \cdot 4,0) = -0,49$ kN

$y_a = -0,36 + (-0,49) \cdot 0,14 = -0,43$ kN

$V_b = 0,5 - 0,51 \cdot 0,14 = 0,43$



$\delta_y = -\Sigma \bar{C}_i \cdot c_i = -(0,43) \cdot 0,01 = -0,00043$ m $= -4,3$ mm