7

СТАТИКА КОНСТРУКЦИЈА

Модул: Хидротехника и водно инжењерство околине, Саобраћајнице, Архитектонско инжењерство

– материјал за вежбе –

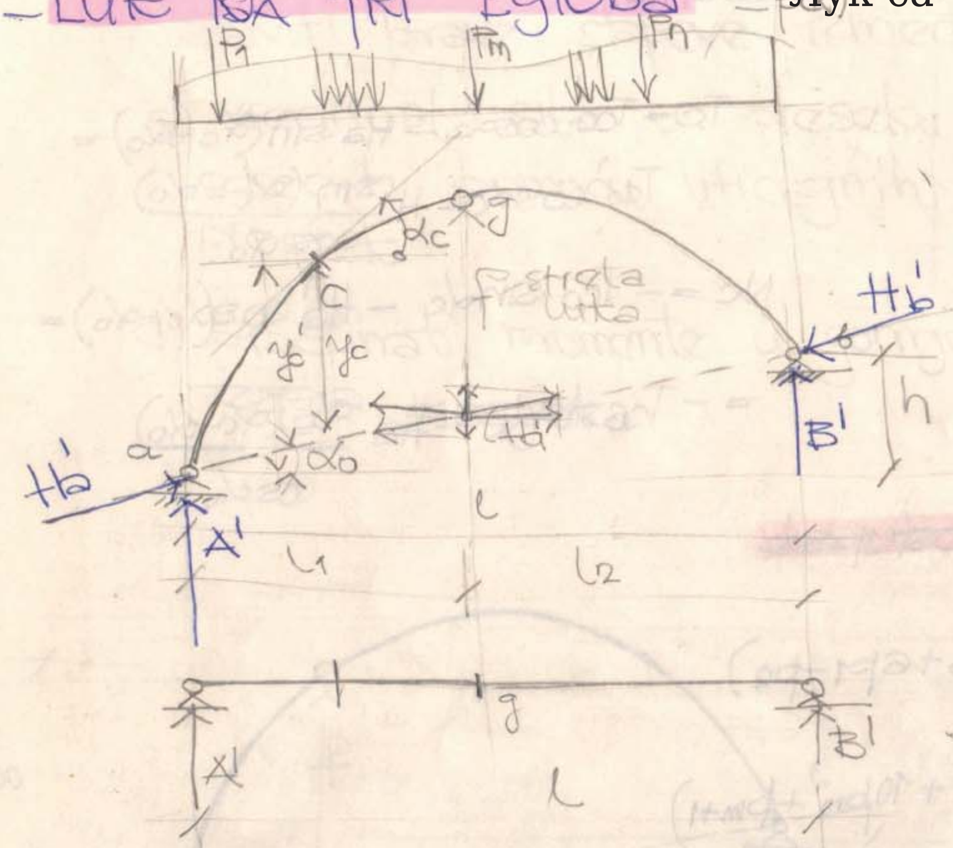
2024.

27.03.1991

NOSAČI KOJI SE SASTOJE OD DVIJE KINEMATIČKI KRUTE PLOČE

– Luk sa tri zgloba

Лук са три зглоба



H_b' pravac lučne sile

$$\left.\begin{matrix}\Sigma M_b \\ \Sigma M_a\end{matrix}\right\} \rightarrow \begin{matrix}A' = A_0 \\ B' = B_0\end{matrix}$$

moment proste grede

$$\left.\begin{matrix}\Sigma M_g^L = 0 \\ \Sigma M_g^d = 0\end{matrix}\right\} \Rightarrow M_{g0}^L - H_a' \cos\alpha_0 \cdot f = 0$$

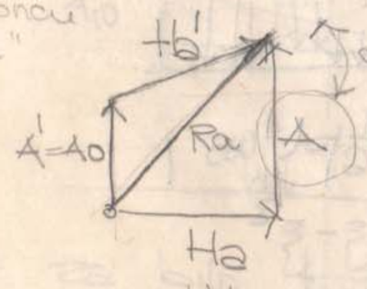
$$M_{g0}^D - H_b' \cos\alpha_0 \cdot f = 0$$

$$H_a = \frac{M_{g0}^L}{f}$$

$$H_b = \frac{M_{g0}^d}{f}$$

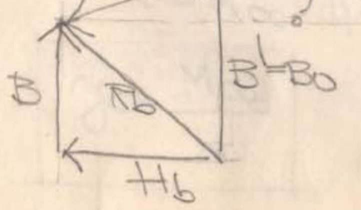
$H_a = H_b = H$ – ako imamo vertikalno opterećenje

u osloncu „a"



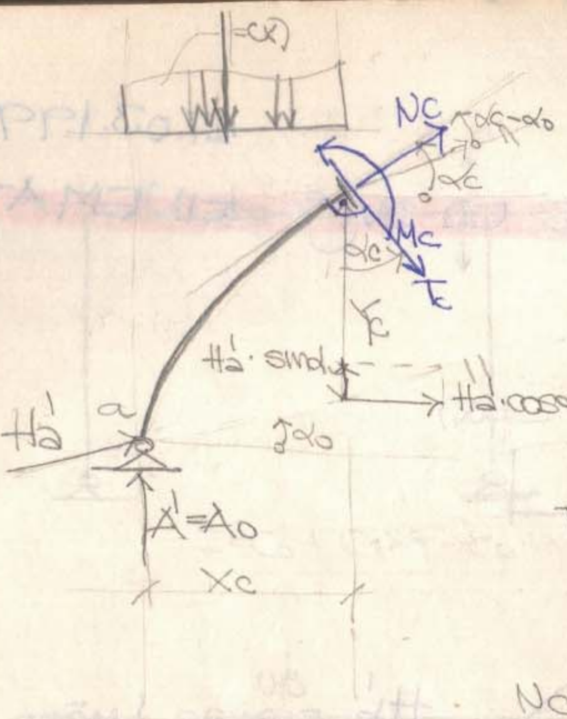
$$H_a = H_a' \cos\alpha$$

$$A = A_0 + H_a' \cdot \sin\alpha_0 = A_0 + H_a \cdot tg\alpha_0$$



$$H_b = H_b' \cos\alpha_0$$

$$B = B_0 - H_b' \cdot \sin\alpha_0 = B_0 - H_b \cdot tg\alpha_0$$



mom. proste grede

momenat

$$M_c = M_{co} - H_a \cdot y_c = \boxed{M_{co} - H y_c}$$

$H_a = H$ po nadaje

$$T_c = T_{co}\cos\alpha_c - H_a' \sin(\alpha_c - \alpha_0) = T_{co} \cdot \cos\alpha_c - H \frac{\sin(\alpha_c - \alpha_0)}{\cos\alpha_0}$$

$$N_c = -T_{co} \cdot \sin\alpha_c - H_a' \cos(\alpha_c - \alpha_0) = -T_{co}\sin\alpha_c - H_a \frac{\cos(\alpha_c - \alpha_0)}{\cos\alpha_0}$$

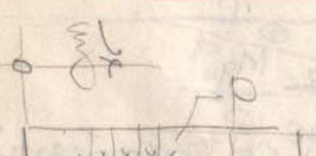
a) numerički postupak

$$P_0 = \frac{\lambda}{24}(7p_0 + 6p_1 - p_2)$$

$$P_m = \frac{\lambda}{12}(p_{m-1} + 10p_m + p_{m+1})$$

$$P_n = \frac{\lambda}{24}(7p_n + 6p_{n-1} - p_{n-2})$$

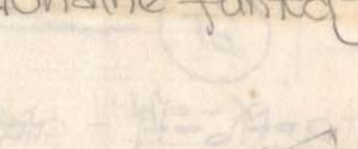
b) analitički postupak (bezdimenzionalne funkcije)



$$\xi = \frac{x}{l} \quad \xi' = \frac{x'}{l}$$

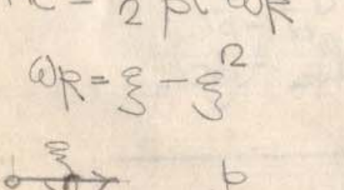
$$M_c = \frac{1}{2} p l^2 \omega_R$$

$$\omega_R = \xi - \xi^2$$



$$M_c = \frac{1}{6} p l^2 \omega_D$$

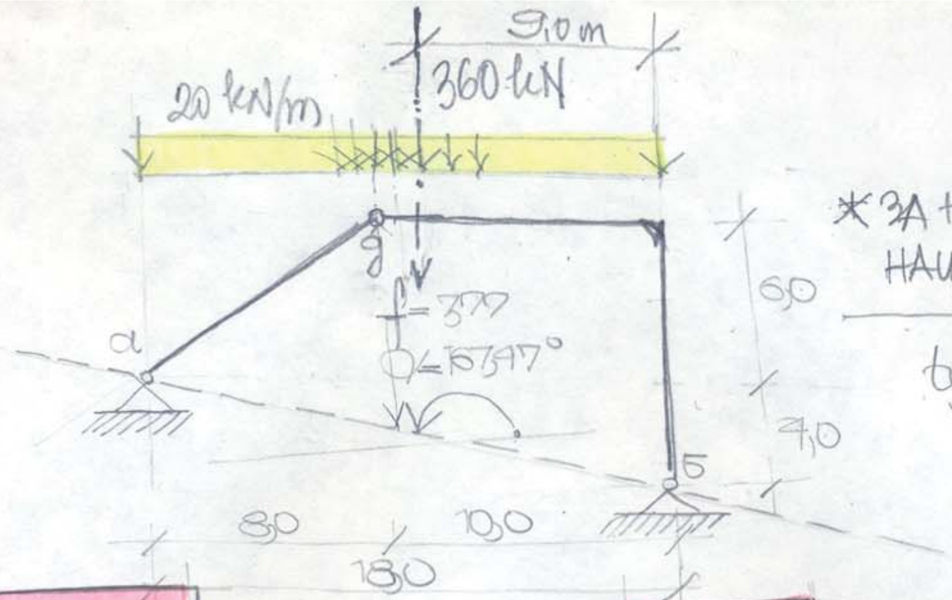
$$\omega_D = \xi - \xi^3$$



$$\xi = 0;\ 0{,}1;\ 0{,}2;\ \ldots;\ 1{,}0$$

$$M_c = \frac{1}{3} p l^2 \omega_p''$$

$$\omega_p'' = \xi - 2\xi^3 + \xi^4$$



* ЗА НОСАЧ СА ОПТЕРЕЋЕЊЕМ НАЦРТАТИ М–ДИЈАГРАМ.

$tg\,\alpha = -0,22$

$\boxed{\sum M_б = 0}$

$A' = \frac{1}{18,0}\left[360 \cdot 9,0\right] = 180\ kN$

$\boxed{\sum M_a = 0}$

$B' = \frac{1}{18}\left[360 \cdot 9,0\right] = 180,0\ kN$

$\boxed{\sum M_g^{L} = 0}$

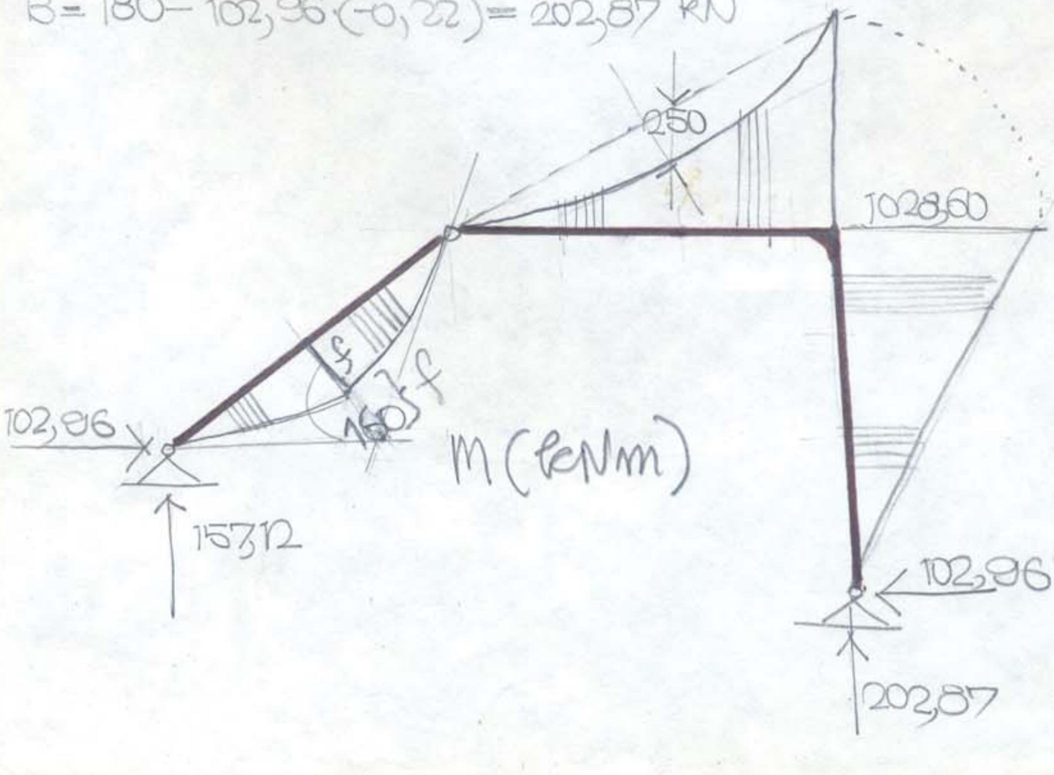
$H_a = \frac{1}{7,77}\left[180 \cdot 8,0 - 20 \cdot 8,0 \cdot 4,0\right] = 102,96\ kN$

$\boxed{\sum M_g^{D} = 0}$

$H_b = \frac{1}{7,77}\left[180 \cdot 10 - 20 \cdot 10 \cdot 5,0\right] = 102,96\ kN$

$A = 180 + 102,96 \cdot (-0,22) = 157,12\ kN$

$B = 180 - 102,96 \cdot (-0,22) = 202,87\ kN$

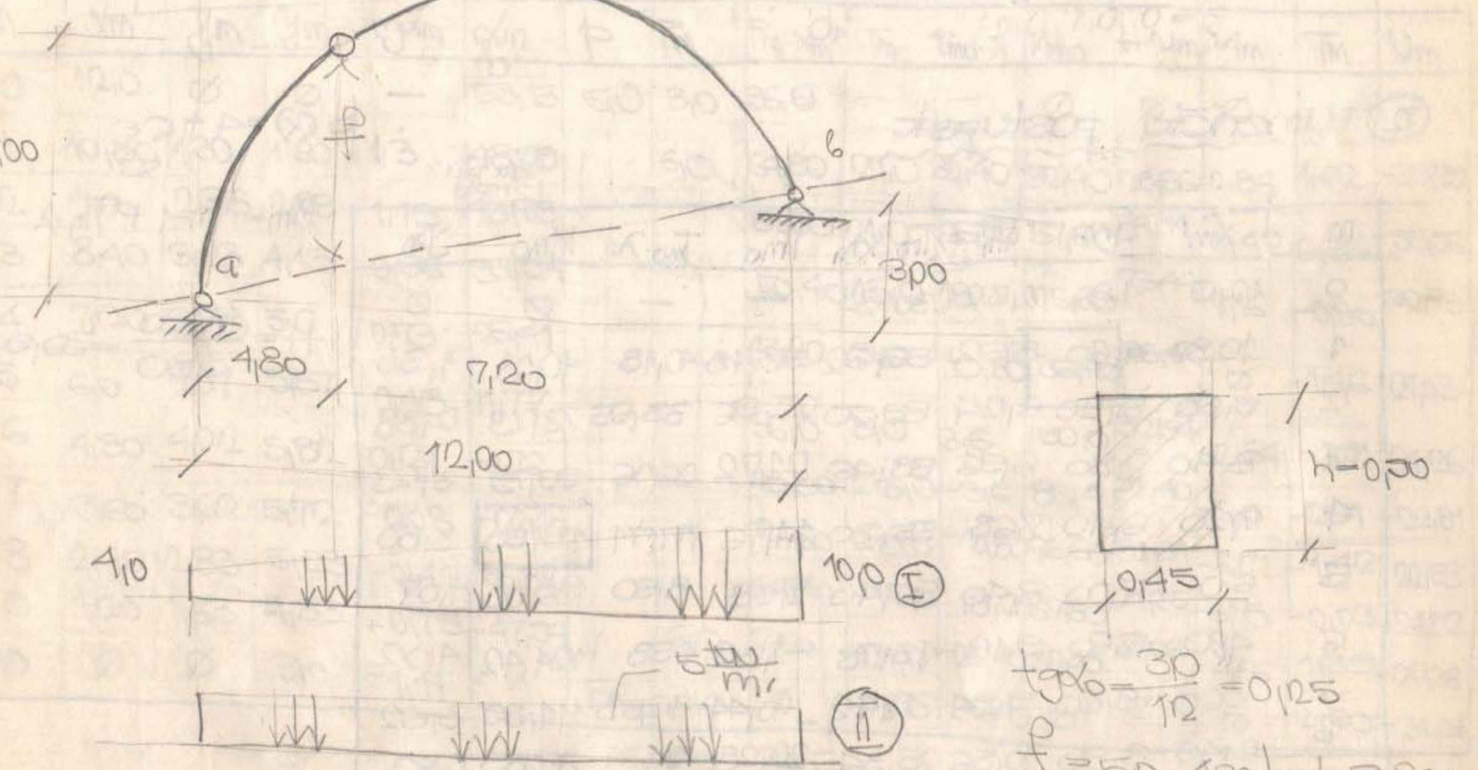


ZADATAK: Za luk na tri zgloba:

a) naći potpornu liniju (racionalnu osu) sa ordinatama u desetinama raspona za opterećenje I

b) za opterećenje II nacrtati dijagrame sila u presecima u desetinama raspona (predpostaviti prave štapove između tačaka)

d) Sračunati momente u gornjoj i donjoj tački jezgra presjeka



$tg\alpha = \frac{3,0}{12} = 0,125$

$f = 5,0 - 4,80\, tg\alpha = 3,80\, m$

$\xi_G = \frac{4,80}{12} = 0,4$

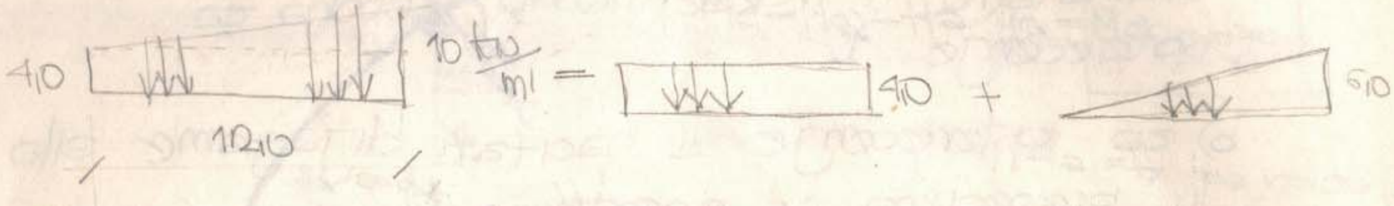
a) za bilo ~~koji~~ presjek rac. osa

da $M_c = M_{co} - H y_c = 0$

$$\boxed{y_c = \frac{M_{co}}{H}}$$

ordinata racionalne ose

① pomoću bezdim. funkcija



$$M_{co} = \frac{1}{2} p l^2 \omega_R + \frac{1}{6} p l^2 \omega_D = \frac{1}{2} 4{,}0 \cdot 12{,}0^2 \cdot (\xi - \xi^2) + \frac{1}{6} 6{,}0 \cdot 12{,}0^2 (\xi - \xi^3) =$$

$$M_{co} = (36\xi - 24\xi^2 - 12\xi^3) \cdot 12$$

potisak luka – $H = \frac{M_{go}}{f} = \frac{M_{co}(\xi = 0{,}4)}{3{,}80} = \frac{(36 \cdot 0{,}4 - 24 \cdot 0{,}4^2 - 12 \cdot 0{,}4^3) \cdot 12}{3{,}80} =$

$$\boxed{H = 30{,}92 \text{ kN}}$$

$\boxed{T_{mo}}$ transferzalna sila proste grede

$$T_{m+1} = T_m - P_m$$

$$y_c = \frac{12}{30{,}92}(36\xi - 24\xi^2 - 12\xi^3)$$

$\xi = 0;\ 0{,}1;\ 0{,}2;\ \ldots;\ 1{,}0$

② numerička postupak

$p(x) = 4 + 6\xi$

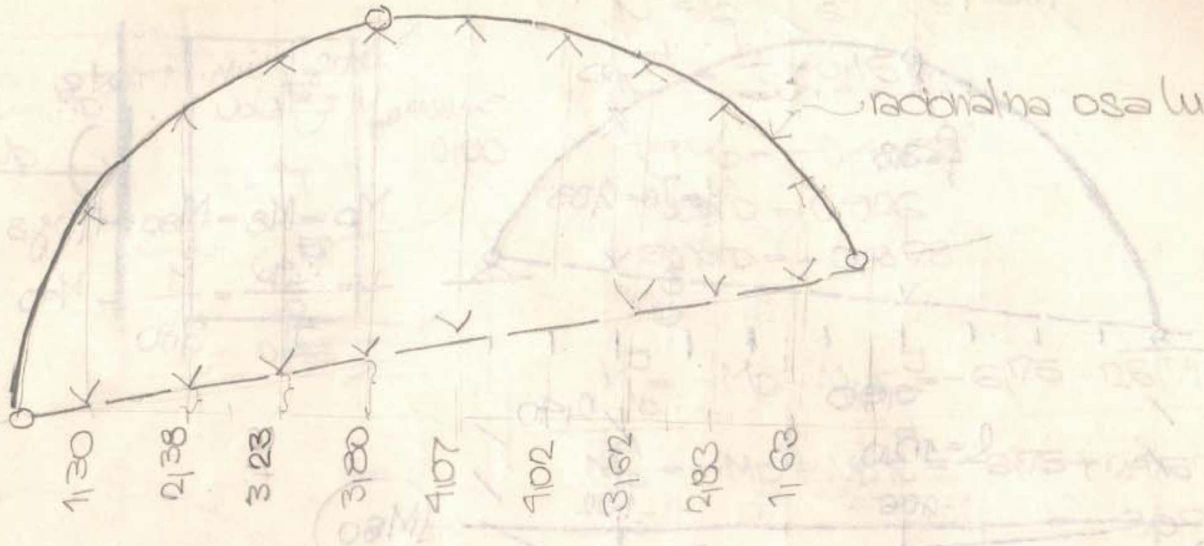
$M_m = M_{m-1} + T_m \cdot \lambda$

$H = \frac{117{,}50}{3{,}80} = 30{,}92$

m	x_m'	p_m	P_m	$P_m \cdot x_m'$	T_{mo}	$T_{mo} \cdot \lambda_i$	M_{mo} (mom. proste grede)	y_m
0	12,0	4,0	2,52	30,24	—	—	∅	∅
1	10,80	4,60	5,52	59,62	33,48	40,18	40,18	1,30
2	9,60	5,20	6,24	59,90	27,96	33,55	73,73	2,38
3	8,40	5,80	6,96	58,46	21,72	26,06	99,79	3,23
4	7,20	6,40	7,68	55,30	14,76	17,71	$\boxed{117{,}50}$	3,80
5	6,0	7,00	8,40	50,40	7,08	8,50	126,00	4,07
6	4,80	7,60	9,12	43,78	−1,32	1,58	124,42	4,02
7	3,60	8,20	9,84	35,42	−10,44	−12,53	111,89	3,62
8	2,40	8,80	10,56	25,34	−20,28	−24,34	87,55	2,83
9	1,20	9,40	11,28	13,54	−30,84	−37,01	50,54	1,63
10	0	10,0	5,88	0	−42,12	−50,54	∅	∅
			84,00	432,0				

$$A_0 = \frac{\sum P_m x_m'}{l} = \frac{432{,}0}{12} = 36{,}0 \text{ kN}$$

b)



$$M_m = M_{m0} - H \cdot y_m$$

$$T_m = (T_{m0} + H \cdot tg\alpha_0) \cdot \cos\alpha_m - H \cdot \sin\alpha_m$$

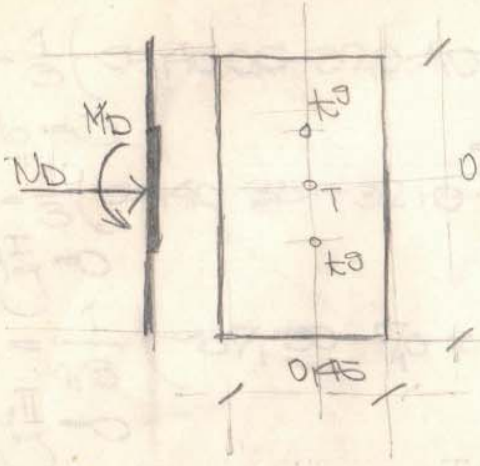
$$N_m = -(T_{m0} + H \cdot tg\alpha_0) \cdot \sin\alpha_m - H \cdot \cos\alpha_m$$

m	x_m'	y_m	y_m'	$tg\alpha_m$	α_m°	P	P_m	$P_m \cdot x_m'$	T_{m0}	$T_{m0} \cdot \lambda$	M_{m0}	$H \cdot y_m$	M_m	T_m	N_m
0	12,0	Ø	Ø	—	53,13	5,0	3,0	36,0	—	—	Ø	Ø	Ø	—	—
1	10,80	1,30	1,60	$1,\dot{3}$	48,99		6,0	64,80	27,0	32,40	32,40	29,56	2,84	1,42	-39,79
2	9,60	2,38	2,98	1,15	43,78			57,60	21,0	25,20	57,60	54,11	3,49	0,35	-35,06
3	8,40	3,23	4,13	0,96	35,94			50,40	15,0	18,0	75,60	73,44	2,16	-0,80	-30,93
4	7,20	3,80	5,0	0,73	25,41			43,20	9,0	10,80	$\boxed{86,40}$	86,40	Ø	-1,46	-27,03
5	6,0	4,07	5,57	0,48	11,77			36,0	3,0	3,6	90,0	92,54	-2,54	-1,91	-24,26
6	4,80	4,02	5,82	0,21	-4,76			28,80	-3,0	-3,6	86,40	91,40	-5,0	-2,01	-22,81
7	3,60	3,62	5,72	-0,08	-22,10			21,60	-9,0	-10,80	75,60	82,31	-6,71	-1,42	-22,93
8	2,40	2,83	5,23	-0,41	-36,87			14,40	-15,0	-18,0	57,60	64,35	-6,75	-0,03	-24,57
9	1,20	1,63	4,33	-0,75	-47,34		6,0	7,20	-21,0	-25,20	32,40	37,06	-4,66	1,39	-27,38
10	Ø	Ø	3,0	-1,11		5,0	3,0	Ø	-27	-32,40	Ø	Ø	Ø	2,60	-31,06
							60	360 Σ							

$$y_m' = y_m + 3\xi \qquad tg\alpha_m = \frac{y_m' - y_{m-1}'}{\lambda}$$

$$A_0 = \frac{\sum P_m \cdot x_m'}{l} = 30 kN \qquad H = \frac{M_{g0}}{f} = \frac{86,40}{3,80} = 22,74$$

d)



$k^g = \frac{h}{6} = \frac{0,90}{6} = 0,15 m$

$k^d = \frac{h}{6} = 0,15 m$

$tg\,\alpha_D = -0,4083$

$\cos\alpha_D = 0,926$

$\sin\alpha_D = -0,378$

M_D^g

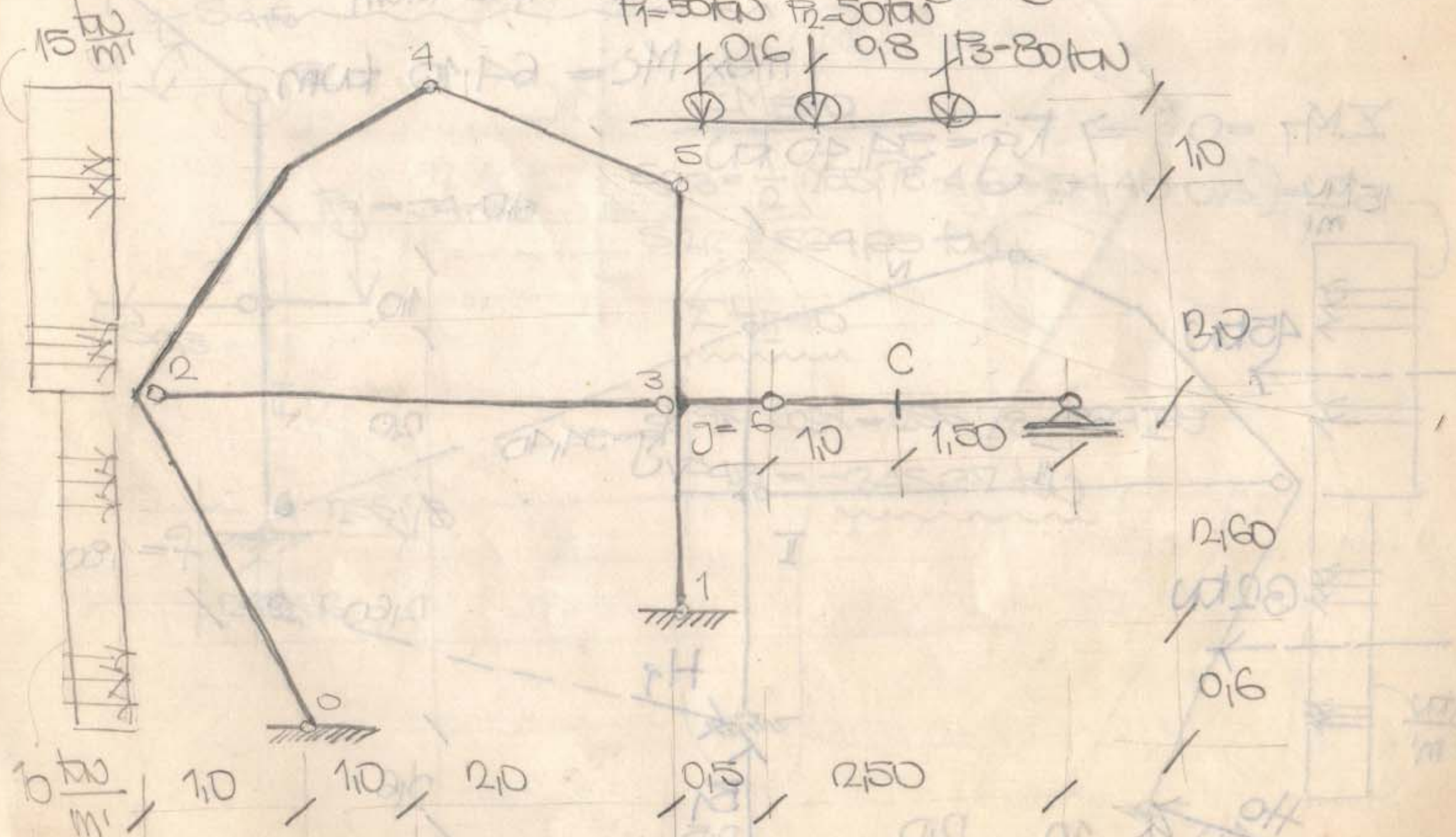
$M_D^g = -M_D - N_D \cdot k^g = -6,75 - 25,71 = -10,44$ tm

$M_D^d = -M_D + N_D k^d = -6,75 + 24,57 \cdot 0,15 =$
$= -3,07$ tm

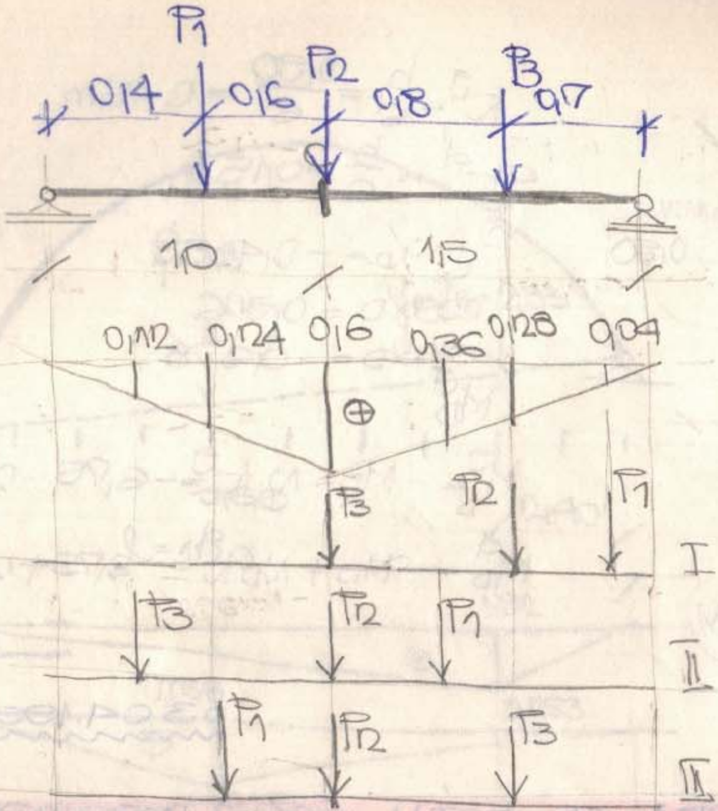
03.04.1991

- NOSAČI SA 3 ZGLOBA OD KOJIH JE 1 IMAGINARAN

1) Za položaj sistema koncentrisanih sila koji na postojećoj gredi izaziva max. moment u presjeku C i pod opterećenjem sa lijeve strane nosača nacrtati dijagram savijanja nosača.



a)



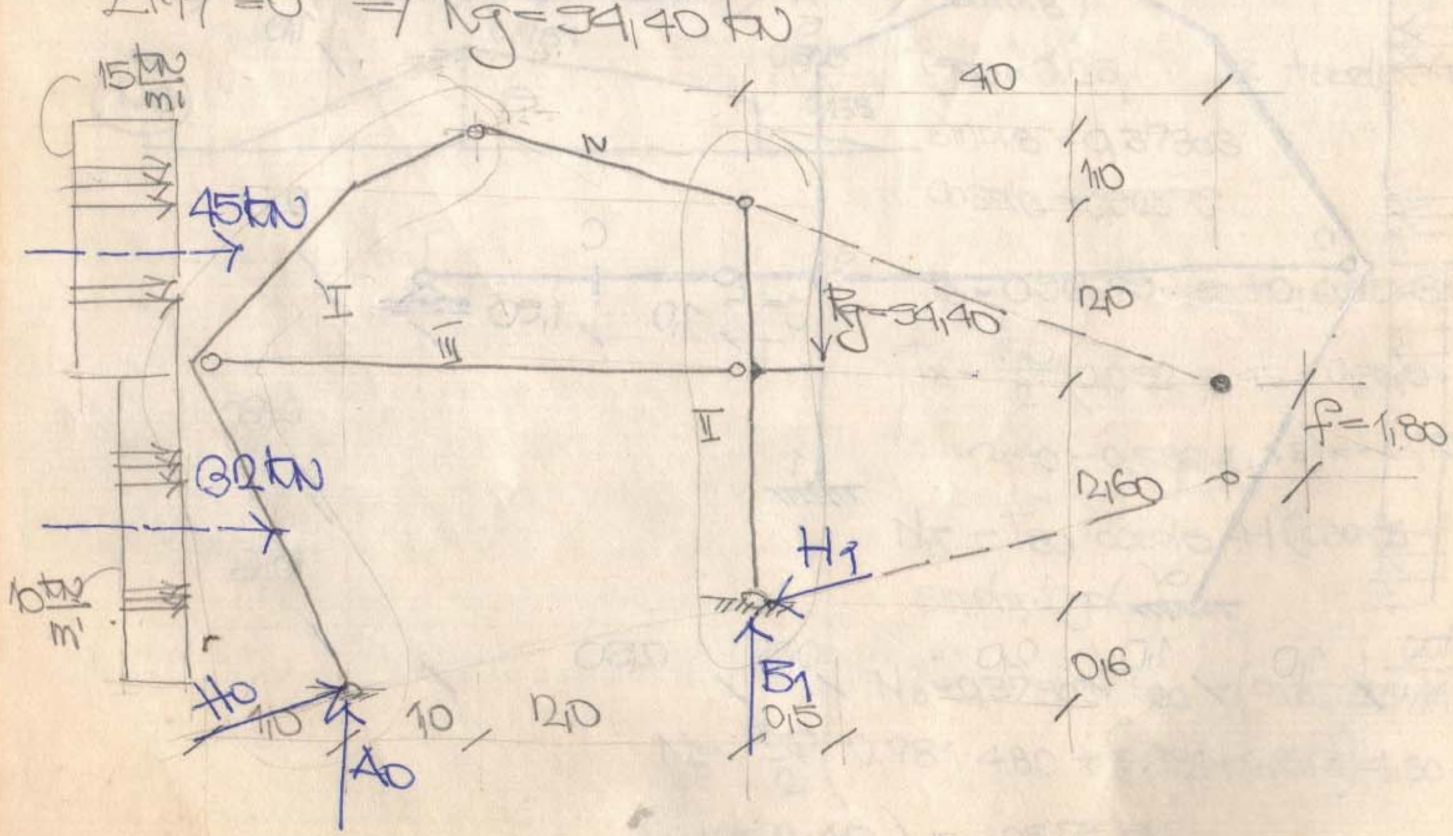
I: $M_C = 80\cdot 0{,}6 + 50(0{,}28 + 0{,}04) = 64$ tnm

II: $M_C = 80\cdot 0{,}12 + 50(0{,}60 + 0{,}36) = 57{,}60$ tnm

III: $M_C = 50(0{,}24 + 0{,}60) + 80\cdot 0{,}28 = 64{,}10$ tnm

max $M_C = 64{,}10$ tnm

$\Sigma M_7 = 0 \Rightarrow R_g = 34{,}40$ tn



$\Sigma M_1 = 0$

$$A_0 = \frac{1}{3}\left(-34{,}40 \cdot 0{,}5 - 32{,}0 \cdot 1{,}10 - 45{,}0 \cdot 4{,}1\right) = -87{,}30 \text{ tn}$$

$\Sigma M_0 = 0$

$$B_1 = \frac{1}{3}\left(34{,}40 \cdot 3{,}5 + 32{,}10 \cdot 1{,}6 + 45{,}0 \cdot 4{,}7\right) = 137{,}70 \text{ tn}$$

$\Sigma M_g^{I} = 0$

$$H_0 = \frac{1}{1{,}8}\left(-87{,}30 \cdot 7{,}0 + 45 \cdot 1{,}5 - 32 \cdot 1{,}60\right) = -332{,}78 \text{ tn}$$

$\Sigma M_g^{II} = 0$

$$H_1 = \frac{1}{1{,}8}\left(-137{,}70 \cdot 4{,}0 + 34{,}40 \cdot 3{,}5\right) = -\cancel{225{,}78} \text{ tn} \quad 255{,}78$$

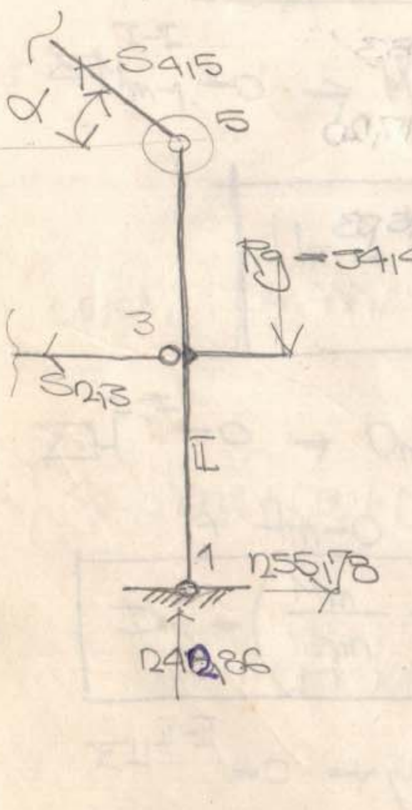
$A = A_0 + H_0 \, tg\alpha_0$

$tg\alpha_0 = 0{,}120$

$A = -87{,}30 - 332{,}78 \cdot 0{,}120 = -154{,}46$ tn

$B = B_0 - H_1 \, tg\alpha_0$

$B = 137{,}70 + 225{,}78 \cdot 0{,}120 = 242{,}86$ tn



$\Sigma M_5 = 0$

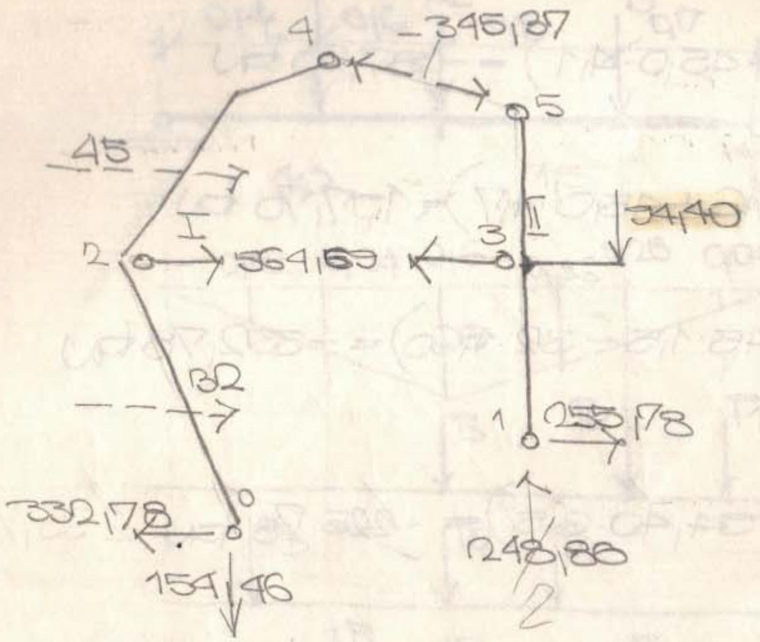
$S_{2,3} = \frac{1}{2}\left(255{,}78 \cdot 4{,}6 - 34{,}40 \cdot 0{,}5\right) =$

$S_{2,3} = 564{,}69$ tn

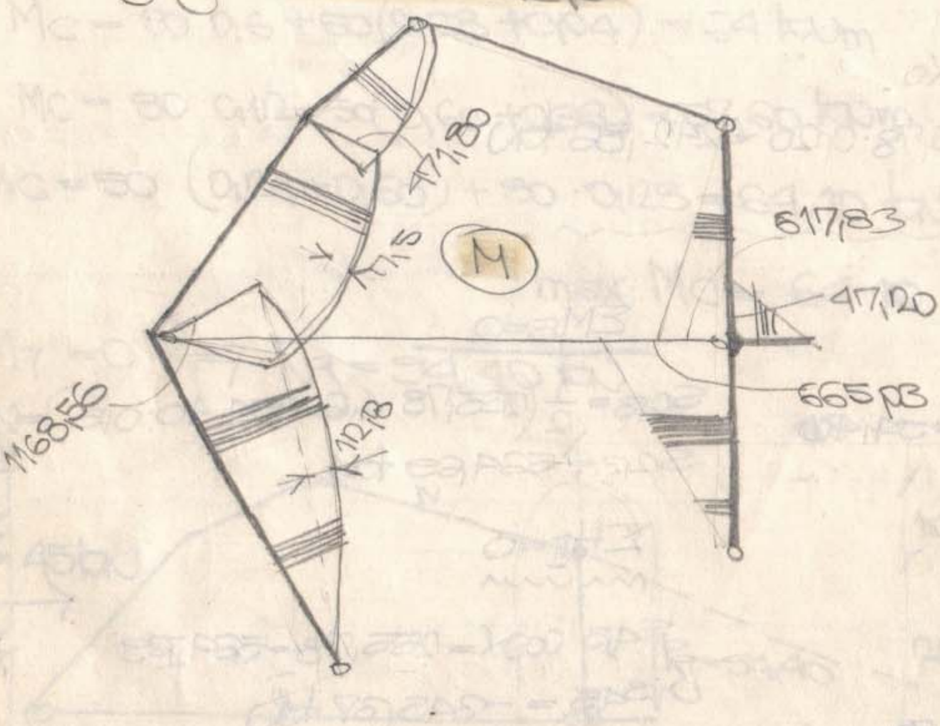
$\Sigma H_{II} = 0$

$S_{4,5} \cdot \cos\alpha = 255{,}78 - 564{,}69$

$S_{4,5} = -345{,}37$ tn

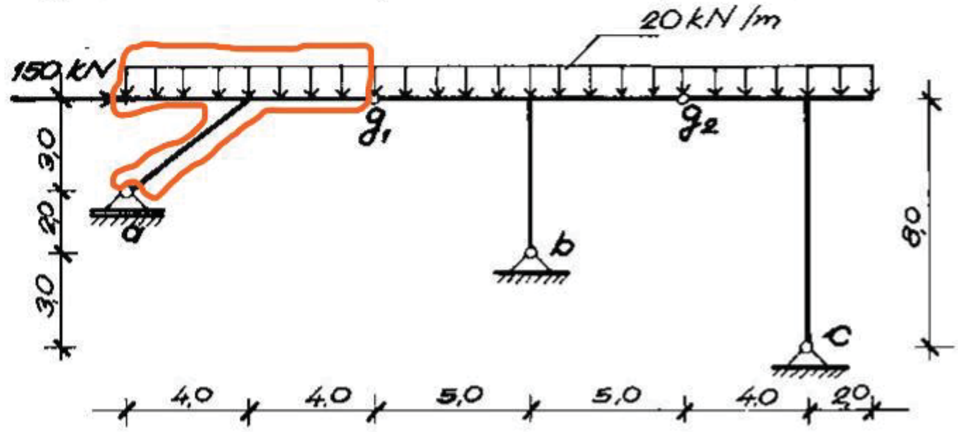


dijagram momenata



Пример

Za dati nosač i opterećenje sračunati i nacrtati dijagrame sila u presecima: M, T i N.



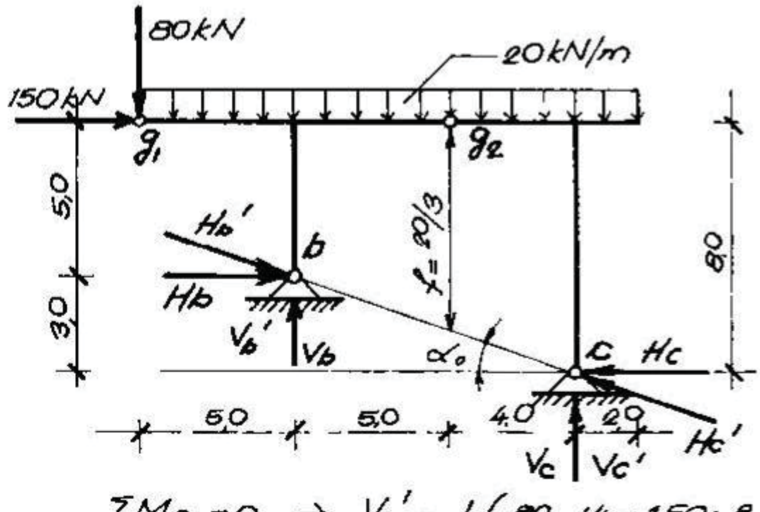
Rešenje:

Određivanje reakcija oslonaca

$$\Sigma M_{g_1}^{l}=0 \Rightarrow V_a \cdot 8{,}0 = 20 \cdot 8{,}0 \cdot 4{,}0 \Rightarrow V_a = 80\ kN$$

$$H_{g1} = 150\ kN \quad ; \quad V_{g1} = 20 \cdot 8{,}0 - 80 = 80\ kN$$

Deo $g_1 - g_2$, a i b predstavlja luk na tri zgloba.



$$tg\,\alpha_0 = \frac{3}{9} = \frac{1}{3}$$

$$f = 8{,}0 - 4{,}0 \cdot \frac{1}{3}$$

$$f = \frac{20}{3}$$

$$\Sigma M_c = 0 \Rightarrow V_b' = \frac{1}{9}\left(80 \cdot 14 - 150 \cdot 8 + 16 \cdot 20 \cdot 6\right) = \frac{1840}{9}$$

$$\Sigma M_b = 0 \Rightarrow V_c' = \frac{1}{9}\left(20 \cdot 16 \cdot 3 + 150 \cdot 5 - 80 \cdot 5\right) = \frac{1310}{9}$$

$$\Sigma M_{g_2}^{l} = 0 \Rightarrow H_b = \frac{1}{f}\left(\frac{1840}{9} \cdot 5 - 80 \cdot 10 - 20 \cdot 10 \cdot 5\right)$$

$$H_b = \frac{3}{20} \cdot \left(-\frac{7000}{9}\right) = -\frac{700}{6}\ kN$$

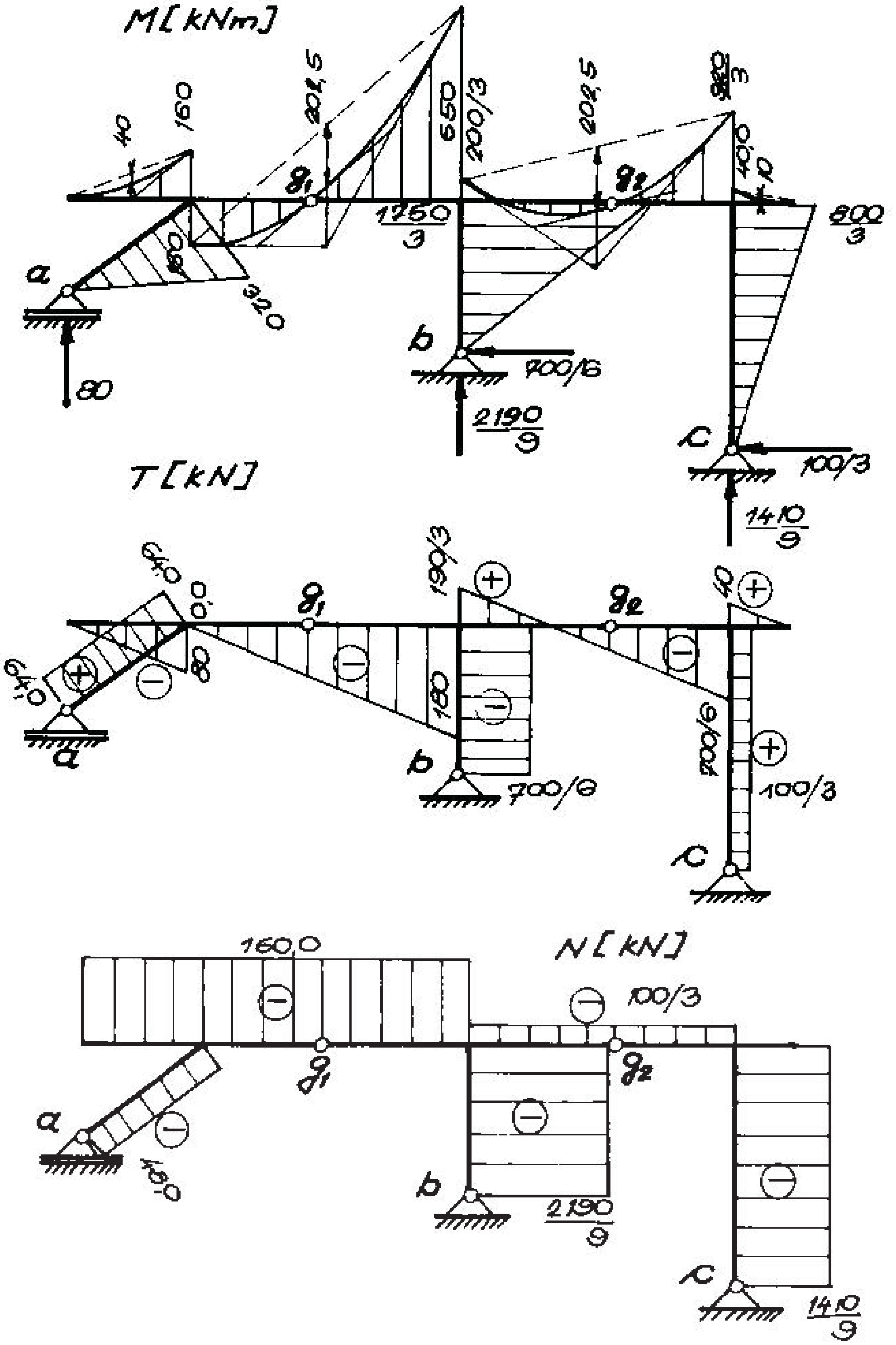
$$\Sigma M_{g_2}^{d} = 0 \Rightarrow H_c = \frac{1}{f}\left(\frac{1310}{9} \cdot 4 - 6 \cdot 20 \cdot 3\right)$$

$$= \frac{3}{20} \cdot \frac{2000}{9} = \frac{100}{3}\ kN$$

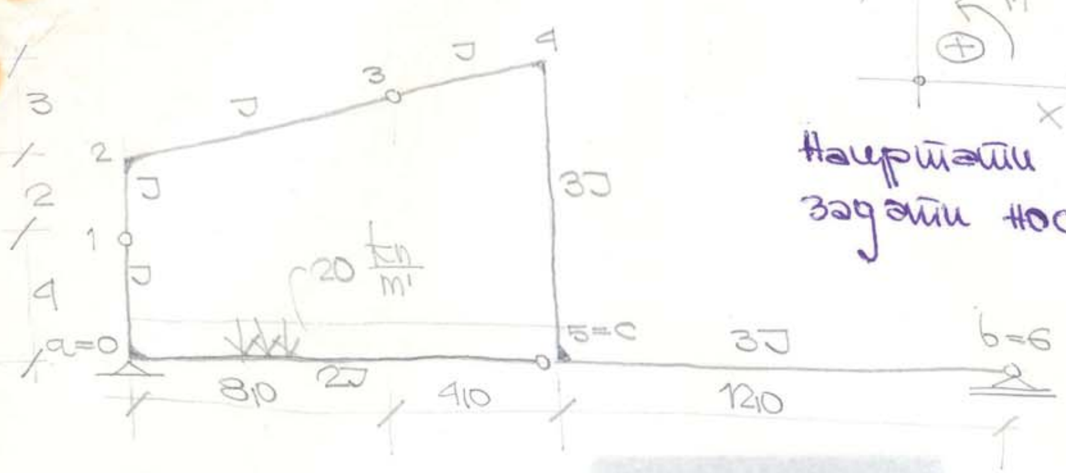
$$V_b = V_b' - H_b \, tg\alpha_0 = \frac{1840}{9} + \frac{700}{6}\cdot\frac{1}{3} = \frac{2190}{9} = 243{,}33\dot{3}\ kN$$

$$V_c = V_c' + H_c \, tg\alpha_0 = \frac{1310}{9} + \frac{100}{3}\cdot\frac{1}{3} = \frac{1410}{9} = 156{,}66\dot{6}\ kN$$

Sa nađenim reakcijama oslonaca lako se dobijaju traženi dijagrami M, T i N.



ЗАДАТАК: (збирка: Никовић, стр.122)

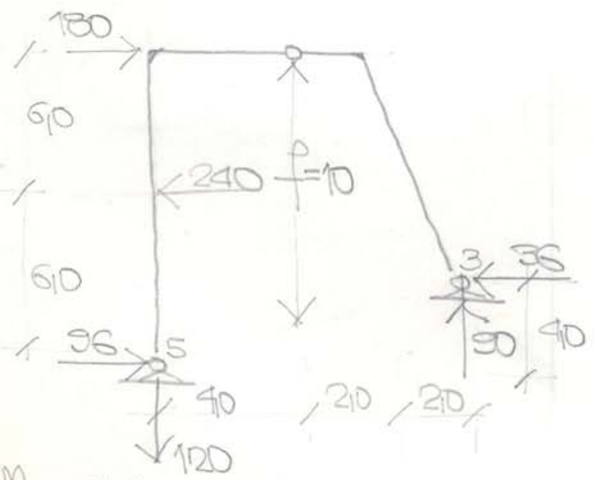


Нацртати (M) дијаграм за задати носач;

$\Sigma M_a = 0$:

$Y_b \cdot 24{,}0 - 240 \cdot 6{,}0 = 0 \qquad Y_b = \frac{240 \cdot 6{,}0}{24{,}0} = 60 \qquad \boxed{Y_b = 60 \text{ tn}}$

$\Sigma M_b = 0$: $\quad \boxed{Y_a = 180 \text{ tn}}$



$\Sigma M_5 = 0$:

$Y_3 \cdot 8{,}0 + 240{,}0 \cdot 6{,}0 - 180 \cdot 12{,}0 = 0$

$Y_3 = \frac{180 \cdot 12{,}0 - 240 \cdot 6{,}0}{8{,}0} = 90$

$\boxed{Y_3 = 90{,}0 \text{ tn}}$

$\Sigma M_3 = 0$:

$-Y_5 \cdot 8{,}0 + 240 \cdot 2{,}0 - 180 \cdot 8{,}0 = 0$

$-Y_5 = \frac{180 \cdot 8{,}0 - 240 \cdot 2{,}0}{8{,}0}$

$-Y_5 = 120 \qquad \boxed{Y_5 = -120 \text{ tn}}$

$\Sigma M_g^l = 0$:

$120 \cdot 4{,}0 - 240 \cdot 6{,}0 + H_5 \cdot 10 = 0$

$H_5 = \frac{240 \cdot 6{,}0 - 120 \cdot 4{,}0}{10}$

$\boxed{H_5 = 96 \text{ tn}}$

$\Sigma M_g^d = 0$:

$90 \cdot 4{,}0 - H_3 \cdot 10 = 0$

$\boxed{H_3 = 36 \text{ tn}}$

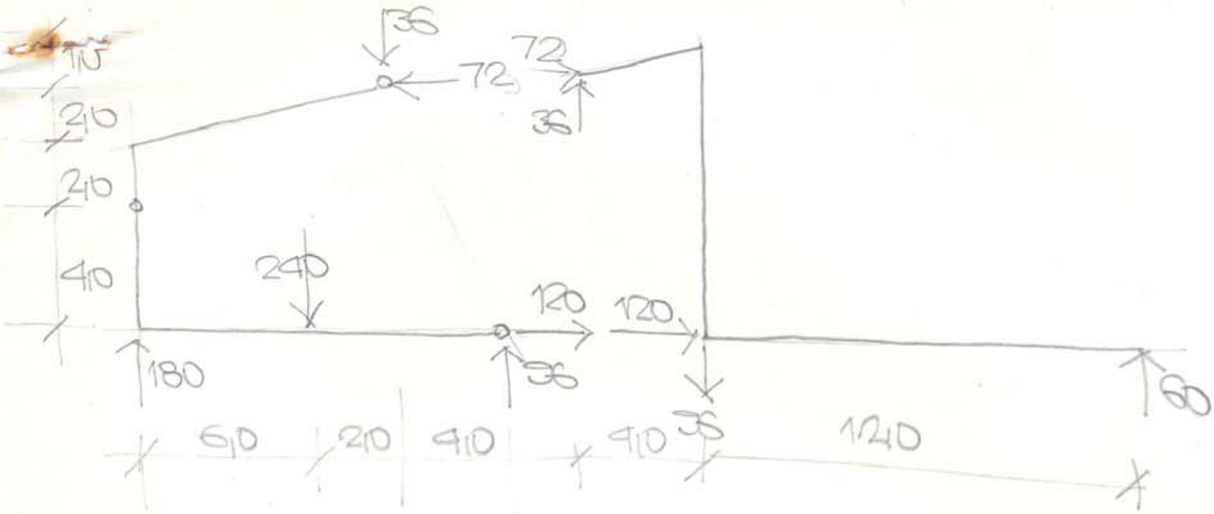
$tg\alpha = 0{,}5$

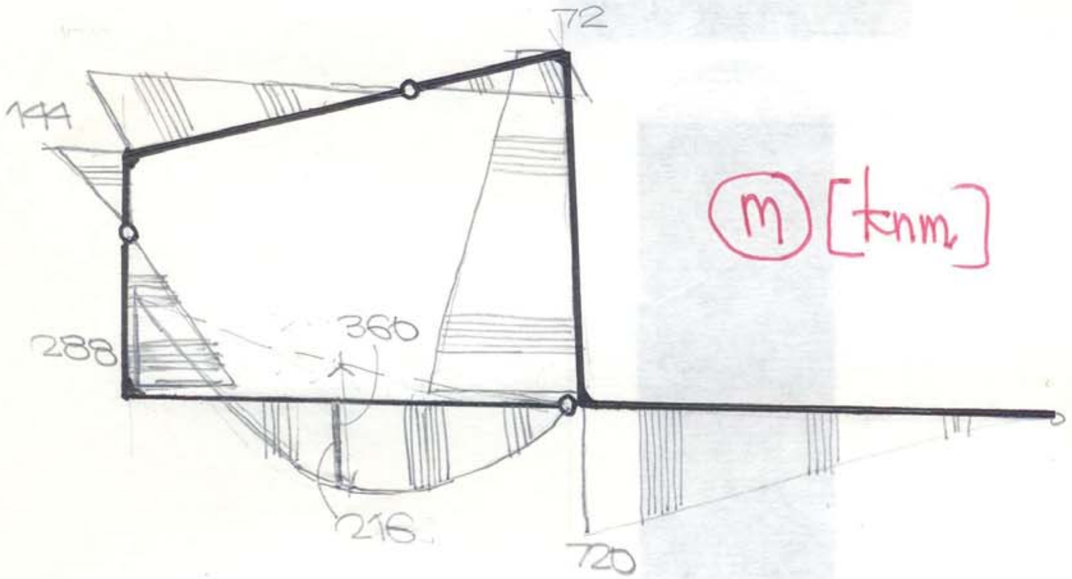
$Y_5 = -120 + 0{,}5 \cdot 96 = -72$

$\boxed{Y_5 = -72{,}0 \text{ tn}}$

$Y_3 = 90 - 0{,}5 \cdot 36 = 72$

$\boxed{Y_3 = 72{,}0 \text{ tn}}$


36
72
72
36
1,0
2,0
2,0
4,0
240
120
120
180
36
36
60
6,0
2,0
4,0
4,0
12,0


72
144
288
360
216
720
m [kNm]