

# 2

## СТАТИКА КОНСТРУКЦИЈА

Модул: Хидротехника и водно инжењерство околине, Саобраћајнице, Архитектонско инжењерство

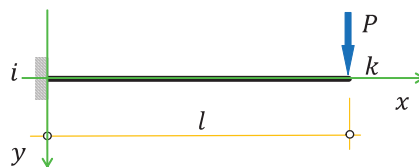
- материјал за вежбе -

2024.

## Примери

За све носаче са датим оптерећењима приказани на скицама потребно је извести функције: еластичне линије, обртања, момената савијања, вертикалних сила попречних пресека дуж носача по теорији првог реда.

Štap tipa - S - KONZOLA



$$v(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

$$\frac{d}{dx}v(x) \rightarrow \alpha_2 + 3 \cdot \alpha_4 \cdot x^2 + 2 \cdot \alpha_3 \cdot x + \frac{p \cdot x^3}{6 \cdot EI} \quad \dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2}v(x) \rightarrow 2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot x + \frac{p \cdot x^2}{2 \cdot EI} \quad \dots M(x) \text{ (kNm)}$$

$$\frac{d^3}{dx^3}v(x) \rightarrow 6 \cdot \alpha_4 + \frac{p \cdot x}{EI} \quad \dots V(x) \text{ (kN)}$$

Konturni uslovi:

$$v(0) = 0$$

$$\phi(0) = 0$$

$$M(L) = 0$$

$$V(L) = -P$$

Given  $p := 0$

$$\alpha_1 + \alpha_2 \cdot 0 + \alpha_3 \cdot 0^2 + \alpha_4 \cdot (0)^3 + \frac{p \cdot 0^4}{24 \cdot EI} = 0$$

$$\alpha_2 + 3 \cdot \alpha_4 \cdot (0)^2 + 2 \cdot \alpha_3 \cdot 0 + \frac{p \cdot (0)^3}{6 \cdot EI} = 0$$

$$2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot L + \frac{p \cdot L^2}{2 \cdot EI} = 0$$

$$6 \cdot \alpha_4 + \frac{p \cdot L}{EI} = -P$$

$$\text{Find}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \rightarrow \begin{pmatrix} 0 \\ 0 \\ \frac{L \cdot P}{2} \\ -\frac{P}{6} \end{pmatrix}$$

Konstante su:

$$\alpha_1 := 0$$

$$\alpha_2 := 0$$

$$\alpha_3 := \frac{L \cdot P}{2}$$

$$\alpha_4 := -\frac{P}{6}$$

Elastična linija savijanja nosača:

$$v(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

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$$v(x) \text{ simplify} \rightarrow -\frac{P \cdot x^2 \cdot (x - 3 \cdot L)}{6} \quad \dots\dots v(x) \text{ (m)}$$

$$\frac{d}{dx} v(x) \text{ simplify} \rightarrow -\frac{P \cdot x \cdot (x - 2 \cdot L)}{2} \quad \dots\dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2} v(x) \text{ simplify} \rightarrow P \cdot (L - x) \quad \dots\dots M(x) \text{ (kNm)}$$

$$\frac{d^3}{dx^3} v(x) \text{ simplify} \rightarrow -P \quad \dots\dots V(x) \text{ (kN)}$$

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Podaci :

$$x := 0, 0.1 \dots 8 \quad L := 8 \text{ m} \quad P := 10 \text{ kN} \quad EI := 5.46 \cdot 10^4 \text{ kNm}^2$$

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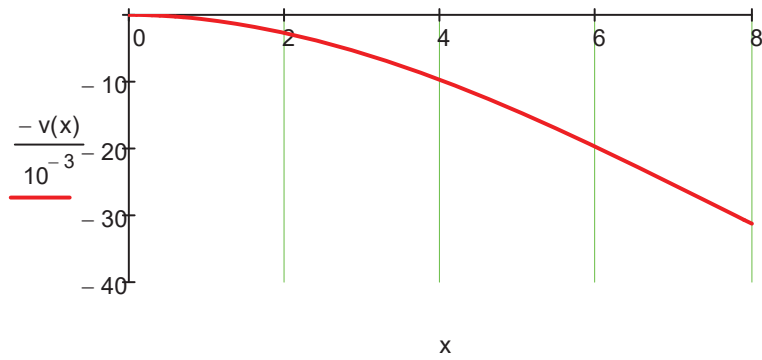
Konačne funkcije uticaja u nosaču:

$$v(x) := -\frac{P \cdot x^2 \cdot (x - 3 \cdot L)}{6} \cdot \frac{1}{EI}$$

$$\phi(x) := -\frac{P \cdot x \cdot (x - 2 \cdot L)}{2} \cdot \frac{1}{EI}$$

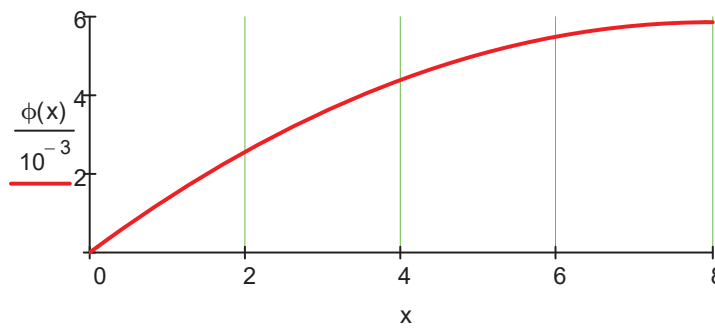
$$M(x) := P \cdot (L - x)$$

$$V(x) := -P$$



(mm)

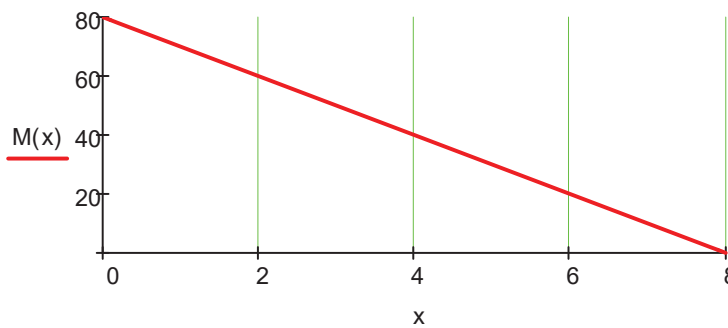
$$\frac{v(L)}{10^{-3}} = 31.26$$



(rad)

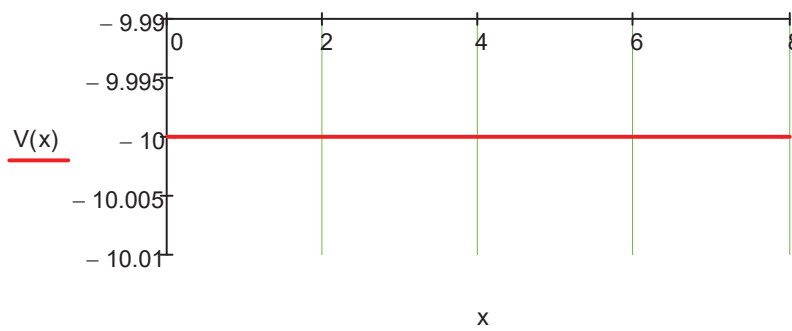
$$\phi(0) = 0$$

$$\phi(L) = 0.00586$$



(kNm)

$$M(0) = 80$$

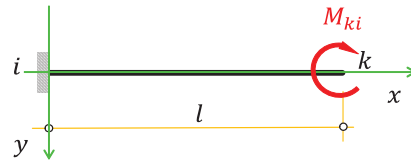


(kN)

$$-V(0) = 10$$

$$V(L) = -10$$

Štap tipa - S - KONZOLA



$$v(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

$$\frac{d}{dx}v(x) \rightarrow \alpha_2 + 3 \cdot \alpha_4 \cdot x^2 + 2 \cdot \alpha_3 \cdot x + \frac{p \cdot x^3}{6 \cdot EI} \quad \dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2}v(x) \rightarrow 2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot x + \frac{p \cdot x^2}{2 \cdot EI} \quad \dots M(x) \text{ (kNm)}$$

$$\frac{d^3}{dx^3}v(x) \rightarrow 6 \cdot \alpha_4 + \frac{p \cdot x}{EI} \quad \dots V(x) \text{ (kN)}$$

Konturni uslovi:

$$v(0) = 0$$

$$\phi(0) = 0$$

$$M(L) = M_{ki}$$

$$V(L) = 0$$

Given

$$p := 0$$

$$\alpha_1 + \alpha_2 \cdot 0 + \alpha_3 \cdot 0^2 + \alpha_4 \cdot (0)^3 + \frac{p \cdot 0^4}{24 \cdot EI} = 0$$

$$\alpha_2 + 3 \cdot \alpha_4 \cdot (0)^2 + 2 \cdot \alpha_3 \cdot 0 + \frac{p \cdot (0)^3}{6 \cdot EI} = 0$$

$$2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot L + \frac{p \cdot L^2}{2 \cdot EI} = M_{ki}$$

$$6 \cdot \alpha_4 + \frac{p \cdot L}{EI} = 0$$

$$\text{Find}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \rightarrow \begin{pmatrix} 0 \\ 0 \\ \frac{M_{ki}}{2} \\ 0 \end{pmatrix}$$

Konstante su:

$$\alpha_1 := 0$$

$$\alpha_2 := 0$$

$$\alpha_3 := \frac{M_{ki}}{2}$$

$$\alpha_4 := 0$$

Elastična linija savijanja nosača:

$$v(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

---

$$v(x) \text{ simplify} \rightarrow \frac{M_{ki} \cdot x^2}{2} \quad \dots\dots v(x) \text{ (m)}$$

$$\frac{d}{dx} v(x) \text{ simplify} \rightarrow M_{ki} \cdot x \quad \dots\dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2} v(x) \text{ simplify} \rightarrow M_{ki} \quad \dots\dots M(x) \text{ (kNm)}$$

$$\frac{d^3}{dx^3} v(x) \text{ simplify} \rightarrow 0 \quad \dots\dots V(x) \text{ (kN)}$$

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Podaci :

$$x := 0, 0.1 \dots 8 \quad \underline{L} := 8 \text{ m} \quad M_{ki} := 10 \text{ kNm} \quad EI := 5.46 \cdot 10^4 \text{ kNm}^2$$

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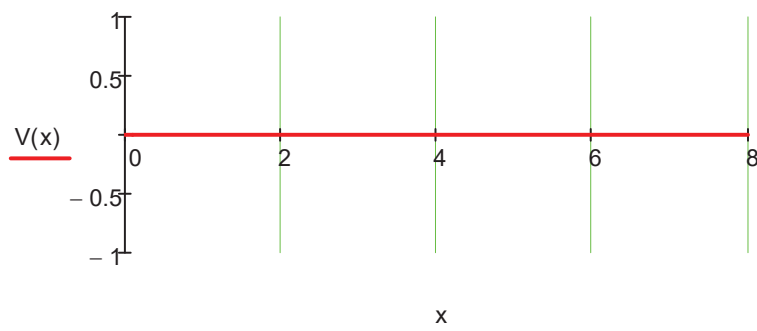
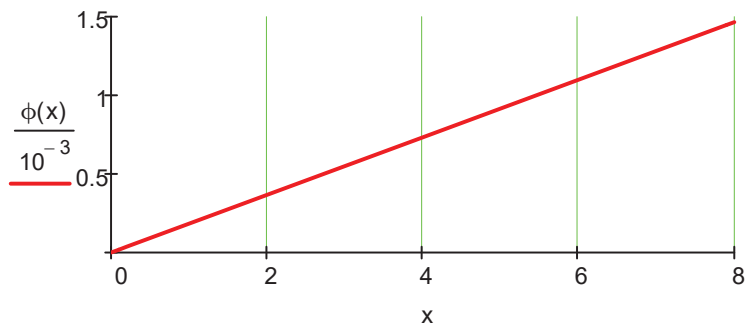
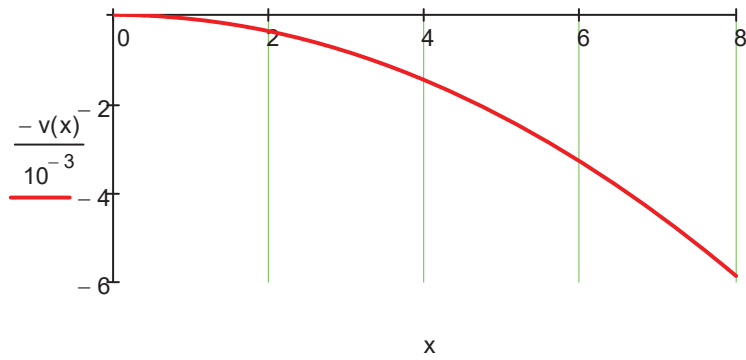
Konačne funkcije uticaja u nosaču:

$$v(x) := \frac{M_{ki} \cdot x^2}{2} \cdot \frac{1}{EI}$$

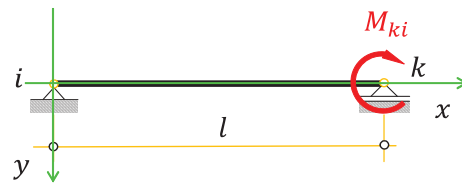
$$\phi(x) := M_{ki} \cdot x \cdot \frac{1}{EI}$$

$$M(x) := M_{ki}$$

$$\underline{V}(x) := 0$$



Štap tipa - PROSTAGREDA



$$v(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

$$\frac{d}{dx}v(x) \rightarrow \alpha_2 + 3 \cdot \alpha_3 \cdot x + 2 \cdot \alpha_4 \cdot x^2 + \frac{p \cdot x^3}{6 \cdot EI} \quad \dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2}v(x) \rightarrow 2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot x + \frac{p \cdot x^2}{2 \cdot EI} \quad \dots M(x) \text{ (kNm)}$$

$$\frac{d^3}{dx^3}v(x) \rightarrow 6 \cdot \alpha_4 + \frac{p \cdot x}{EI} \quad \dots V(x) \text{ (kN)}$$

Konturni uslovi:

$$v(0) = 0$$

$$M(0) = 0$$

$$M(L) = M$$

$$v(L) = 0$$

Given  $p := 0$

$$\alpha_1 + \alpha_2 \cdot 0 + \alpha_3 \cdot 0^2 + \alpha_4 \cdot (0)^3 + \frac{p \cdot 0^4}{24 \cdot EI} = 0$$

$$2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot 0 + \frac{p \cdot 0^2}{2 \cdot EI} = 0$$

$$2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot L + \frac{p \cdot L^2}{2 \cdot EI} = M$$

$$\alpha_1 + \alpha_2 \cdot L + \alpha_3 \cdot L^2 + \alpha_4 \cdot L^3 + \frac{p \cdot L^4}{24 \cdot EI} = 0$$

Find( $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ )  $\rightarrow$   $\begin{pmatrix} 0 \\ L \cdot M \\ -\frac{L \cdot M}{6} \\ 0 \\ M \\ \frac{M}{6 \cdot L} \end{pmatrix}$  Konstante su:

$$\alpha_1 := 0$$

$$\alpha_3 := 0$$

$$\alpha_2 := -\frac{L \cdot M}{6}$$

$$\alpha_4 := \frac{M}{6 \cdot L}$$



Elastična linija savijanja štapa:

$$v(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

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$$v(x) \text{ simplify} \rightarrow -\frac{M \cdot x \cdot (L^2 - x^2)}{6 \cdot L} \quad \dots\dots v(x) \text{ (m)}$$

$$\frac{d}{dx} v(x) \text{ simplify} \rightarrow -\frac{M \cdot (L^2 - 3 \cdot x^2)}{6 \cdot L} \quad \dots\dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2} v(x) \text{ simplify} \rightarrow \frac{M \cdot x}{L} \quad \dots\dots M(x) \text{ (kNm)}$$

$$\frac{d^3}{dx^3} v(x) \text{ simplify} \rightarrow \frac{M}{L} \quad \dots\dots V(x) \text{ (kN)}$$

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Podaci :

$$x := 0, 0.1 \dots 8 \quad \overset{\text{mm}}{L} := 8 \text{ m} \quad M := 10 \text{ kNm} \quad EI := 5.46 \cdot 10^4 \text{ kNm}^2$$

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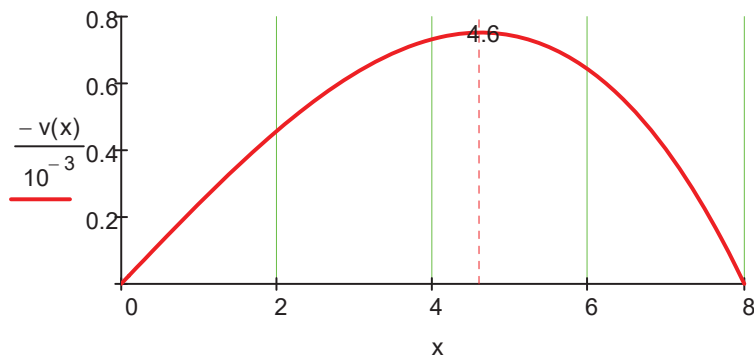
Konačne funkcije uticaja u štapa:

$$v(x) := -\frac{M \cdot x \cdot (L^2 - x^2)}{6 \cdot L} \cdot \frac{1}{EI}$$

$$\phi(x) := -\frac{M \cdot (L^2 - 3 \cdot x^2)}{6 \cdot L} \cdot \frac{1}{EI}$$

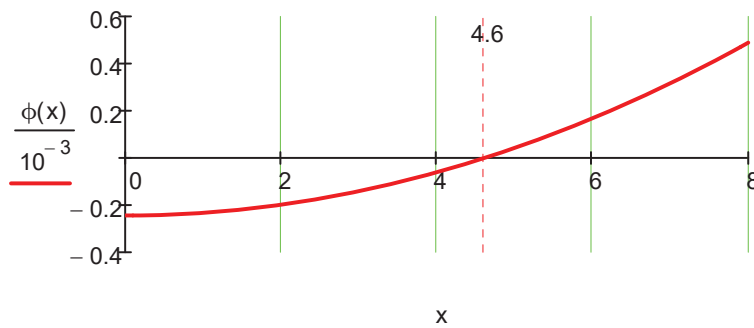
$$\overset{\text{mm}}{M}(x) := \frac{M \cdot x}{L}$$

$$V(x) := \frac{M}{L} \quad \overset{\text{mm}}{V} := \frac{10}{8}$$



(mm)

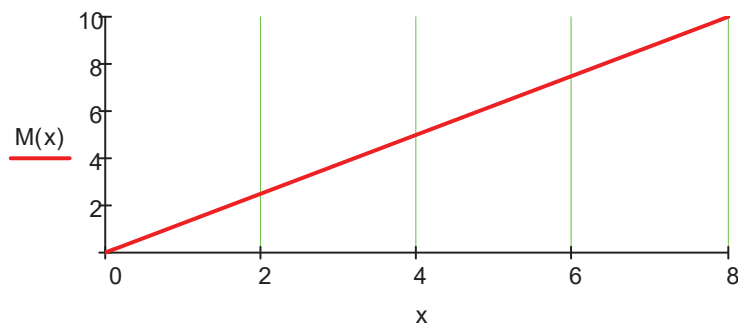
$$\frac{v(4.6)}{10^{-3}} = -0.75$$



(rad)

$$\phi(0) = -0.00024$$

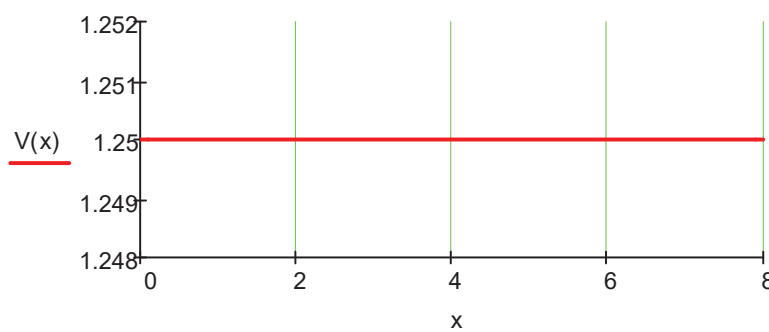
$$\phi(L) = 0.00049$$



(kNm)

$$M(L) = 10$$

$$V(x) := V$$

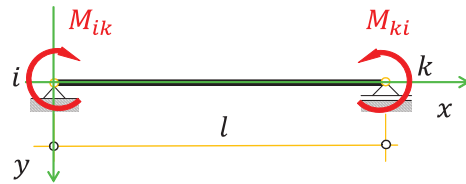


(kN)

$$-V(0) = -1.25$$

$$V(L) = 1.25$$

Štap tipa - PROSTA GREDA - čisto savijanje



$$v(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

$$\frac{d}{dx}v(x) \rightarrow \alpha_2 + 3 \cdot \alpha_3 \cdot x + 2 \cdot \alpha_4 \cdot x + \frac{p \cdot x^3}{6 \cdot EI} \quad \dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2}v(x) \rightarrow 2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot x + \frac{p \cdot x^2}{2 \cdot EI} \quad \dots M(x) \text{ (kNm)}$$

$$\frac{d^3}{dx^3}v(x) \rightarrow 6 \cdot \alpha_4 + \frac{p \cdot x}{EI} \quad \dots V(x) \text{ (kN)}$$

Konturni uslovi:

$$v(0) = 0$$

$$M(0) = M_{ik}$$

$$M(L) = -M_{ki}$$

$$v(L) = 0$$

Given  $p := 0$

$$\alpha_1 + \alpha_2 \cdot 0 + \alpha_3 \cdot 0^2 + \alpha_4 \cdot (0)^3 + \frac{p \cdot 0^4}{24 \cdot EI} = 0$$

$$2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot 0 + \frac{p \cdot 0^2}{2 \cdot EI} = -M_{ik}$$

$$2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot L + \frac{p \cdot L^2}{2 \cdot EI} = M_{ki}$$

$$\alpha_1 + \alpha_2 \cdot L + \alpha_3 \cdot L^2 + \alpha_4 \cdot L^3 + \frac{p \cdot L^4}{24 \cdot EI} = 0$$

$$\text{Find}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \rightarrow \begin{pmatrix} 0 \\ \frac{L \cdot M_{ik}}{3} - \frac{L \cdot M_{ki}}{6} \\ \frac{M_{ik}}{2} \\ \frac{M_{ik} + M_{ki}}{6 \cdot L} \end{pmatrix}$$

Konstante su:

$$\alpha_1 := 0$$

$$\alpha_3 := -\frac{M_{ik}}{2}$$

$$\alpha_2 := \frac{L \cdot M_{ik}}{3} - \frac{L \cdot M_{ki}}{6}$$

$$\alpha_4 := \frac{M_{ik} + M_{ki}}{6 \cdot L}$$

Elastična linija savijanja nosača:

$$v(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

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$$v(x) \text{ simplify} \rightarrow -\frac{x \cdot (L - x) \cdot (L \cdot M_{ki} - 2 \cdot L \cdot M_{ik} + M_{ik} \cdot x + M_{ki} \cdot x)}{6 \cdot L} \quad \dots\dots v(x) \text{ (m)}$$

$$\frac{d}{dx} v(x) \text{ simplify} \rightarrow L \cdot \left( \frac{M_{ik}}{3} - \frac{M_{ki}}{6} \right) + \frac{3 \cdot M_{ik} \cdot x^2 + 3 \cdot M_{ki} \cdot x^2}{6 \cdot L} - M_{ik} \cdot x \quad \dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2} v(x) \text{ simplify} \rightarrow \frac{M_{ik} \cdot x + M_{ki} \cdot x}{L} - M_{ik} \quad \dots\dots\dots M(x) \text{ (kNm)}$$

$$\frac{d^3}{dx^3} v(x) \text{ simplify} \rightarrow \frac{M_{ik} + M_{ki}}{L} \quad \dots\dots\dots V(x) \text{ (kN)}$$

Podaci :

$$x := 0, 0.1 \dots 8 \quad \underline{L} := 8 \text{ m} \quad M_{ik} := 10 \text{ kNm} \quad M_{ki} := -10 \text{ kNm} \quad EI := 5.46 \cdot 10^4 \text{ kNm}^2$$

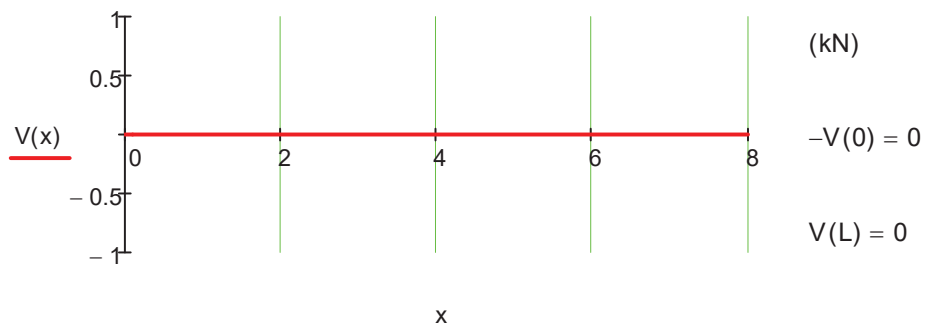
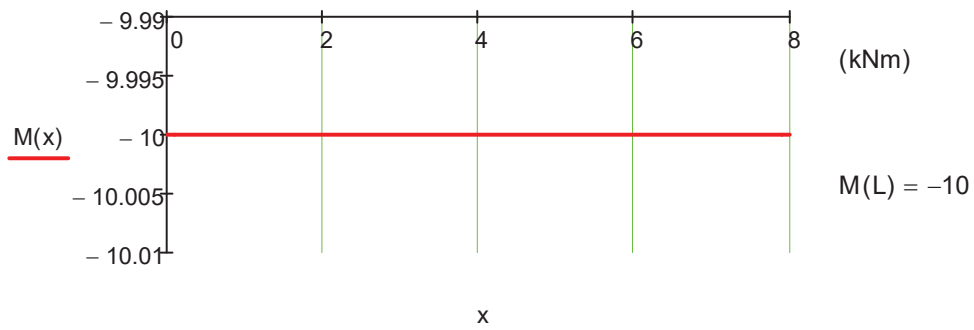
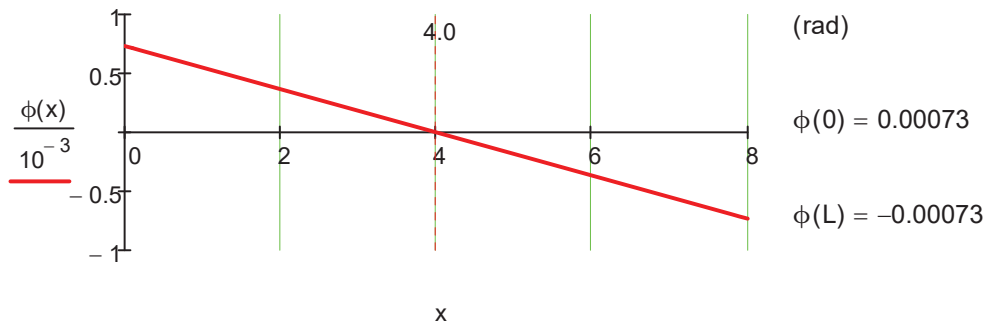
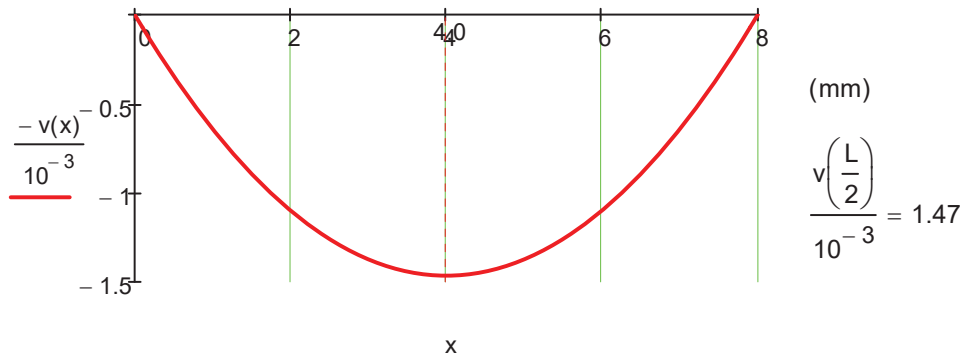
Konačne funkcije uticaja nosača:

$$v(x) := -\frac{x \cdot (L - x) \cdot (L \cdot M_{ki} - 2 \cdot L \cdot M_{ik} + M_{ik} \cdot x + M_{ki} \cdot x)}{6 \cdot L} \cdot \frac{1}{EI}$$

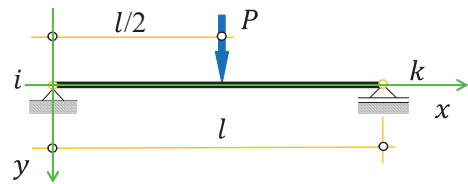
$$\phi(x) := \left[ L \cdot \left( \frac{M_{ik}}{3} - \frac{M_{ki}}{6} \right) + \frac{3 \cdot M_{ik} \cdot x^2 + 3 \cdot M_{ki} \cdot x^2}{6 \cdot L} - M_{ik} \cdot x \right] \cdot \frac{1}{EI}$$

$$M(x) := \frac{M_{ik} \cdot x + M_{ki} \cdot x}{L} - M_{ik}$$

$$\underline{V}(x) := \frac{M_{ik} + M_{ki}}{L}$$



Štap tipa - PROSTA GREDA



$$v(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

$$\frac{d}{dx}v(x) \rightarrow \alpha_2 + 3 \cdot \alpha_3 \cdot x + 2 \cdot \alpha_4 \cdot x^2 + \frac{p \cdot x^3}{6 \cdot EI} \quad \dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2}v(x) \rightarrow 2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot x + \frac{p \cdot x^2}{2 \cdot EI} \quad \dots M(x) \text{ (kNm)}$$

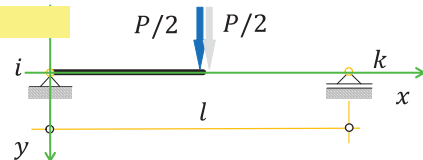
$$\frac{d^3}{dx^3}v(x) \rightarrow 6 \cdot \alpha_4 + \frac{p \cdot x}{EI} \quad \dots V(x) \text{ (kN)}$$

Konturni uslovi leve polovine nosača:

$$v(0) = 0 \quad M(0) = 0$$

$$v\left(\frac{L}{2}\right) = \frac{-P}{2}$$

$$\phi\left(\frac{L}{2}\right) = 0$$



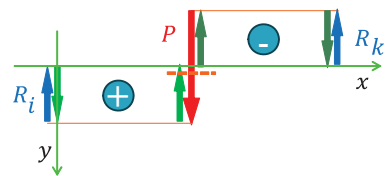
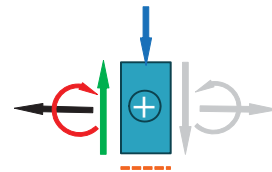
Given  $p := 0$

$$\alpha_1 + \alpha_2 \cdot 0 + \alpha_3 \cdot 0^2 + \alpha_4 \cdot (0)^3 + \frac{p \cdot 0^4}{24 \cdot EI} = 0$$

$$2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot 0 + \frac{p \cdot 0^2}{2 \cdot EI} = 0$$

$$6 \cdot \alpha_4 + \frac{p \cdot \frac{L}{2}}{EI} = \frac{-P}{2}$$

$$\alpha_2 + 3 \cdot \alpha_3 \cdot \left(\frac{L}{2}\right)^2 + 2 \cdot \alpha_4 \cdot \frac{L}{2} + \frac{p \cdot \left(\frac{L}{2}\right)^3}{6 \cdot EI} = 0$$



$$\text{Find}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \rightarrow \begin{pmatrix} 0 \\ \frac{L^2 \cdot P}{16} \\ 0 \\ \frac{P}{12} \end{pmatrix}$$

Konstante su:

$$\alpha_1 := 0$$

$$\alpha_3 := 0$$

$$\alpha_2 := \frac{L^2 \cdot P}{16}$$

$$\alpha_4 := -\frac{P}{12}$$

Elastična linija savijanja leve polovine nosača:

$$v_L(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

---

$$v_L(x) \text{ simplify} \rightarrow \frac{P \cdot x \cdot (3 \cdot L^2 - 4 \cdot x^2)}{48} \quad \dots\dots v(x) \text{ (m)}$$

$$\frac{d}{dx} v_L(x) \text{ simplify} \rightarrow \frac{P \cdot (L^2 - 4 \cdot x^2)}{16} \quad \dots \phi(x) \text{ (rad)}$$

$$\frac{d^2}{dx^2} v_L(x) \text{ simplify} \rightarrow -\frac{P \cdot x}{2} \quad \dots\dots\dots M(x) \text{ (kNm)}$$

$$\frac{d^3}{dx^3} v_L(x) \text{ simplify} \rightarrow -\frac{P}{2} \quad \dots\dots\dots V(x) \text{ (kN)}$$

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Podaci :

$$x := 0, 0.1 \dots 4 \quad L := 8 \text{ m} \quad P := 10 \text{ kN} \quad EI := 5.46 \cdot 10^4 \text{ kNm}^2$$

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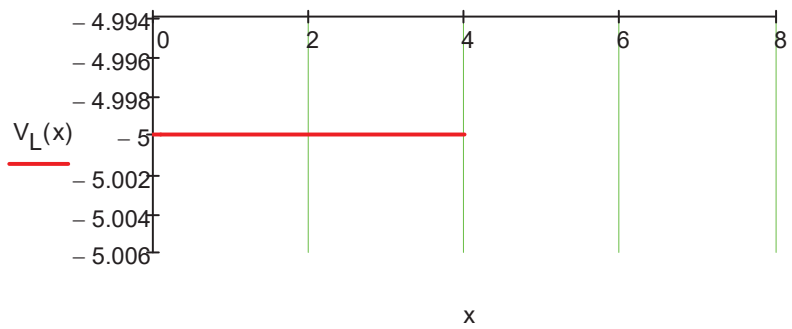
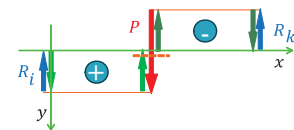
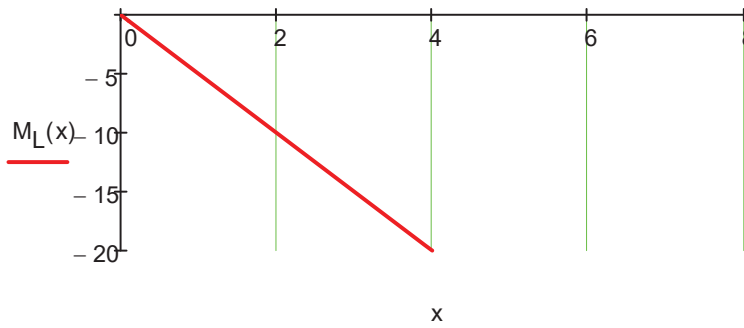
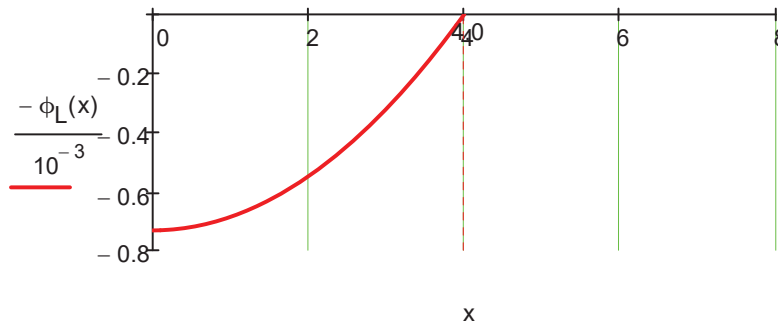
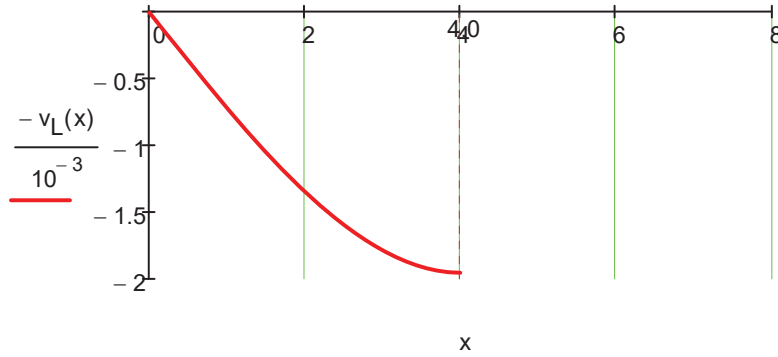
Konačne funkcije uticaja leve polovini nosača:

$$v_L(x) := \frac{P \cdot x \cdot (3 \cdot L^2 - 4 \cdot x^2)}{48} \cdot \frac{1}{EI}$$

$$\phi_L(x) := \frac{P \cdot (L^2 - 4 \cdot x^2)}{16} \cdot \frac{1}{EI}$$

$$M_L(x) := -\frac{P \cdot x}{2}$$

$$V_L(x) := -\frac{P}{2}$$





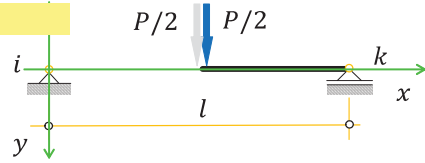
Konturni uslovi desne polovine nosača:

$$v\left(\frac{L}{2}\right) = \frac{P}{2}$$

$$\phi\left(\frac{L}{2}\right) = 0$$

$$v(L) = 0$$

$$M(L) = 0$$



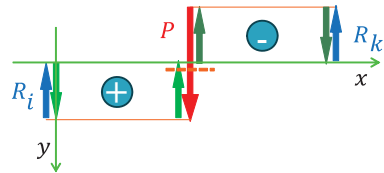
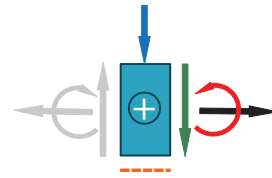
Given  $p := 0$

$$6 \cdot \alpha_4 + \frac{p \cdot \frac{L}{2}}{EI} = \frac{P}{2}$$

$$\alpha_2 + 3 \cdot \alpha_4 \cdot \left(\frac{L}{2}\right)^2 + 2 \cdot \alpha_3 \cdot \frac{L}{2} + \frac{p \cdot \left(\frac{L}{2}\right)^3}{6 \cdot EI} = 0$$

$$\alpha_1 + \alpha_2 \cdot L + \alpha_3 \cdot L^2 + \alpha_4 \cdot (L)^3 + \frac{p \cdot L^4}{24 \cdot EI} = 0$$

$$2 \cdot \alpha_3 + 6 \cdot \alpha_4 \cdot L + \frac{p \cdot L^2}{2 \cdot EI} = 0$$



Konstante su:

$$\alpha_1 := -\frac{L^3 \cdot P}{48}$$

$$\alpha_2 := \frac{3 \cdot L^2 \cdot P}{16}$$

$$\alpha_3 := -\frac{L \cdot P}{4}$$

$$\alpha_4 := \frac{P}{12}$$

Elastična linija desne polovine nosača:

$$v_D(x) := \alpha_1 + \alpha_2 \cdot x + \alpha_3 \cdot x^2 + \alpha_4 \cdot x^3 + \frac{p \cdot x^4}{24 \cdot EI}$$

Konačne funkcije uticaja desne polovine nosača:

$$v_D(x) := -\frac{P \cdot (L - x) \cdot (L^2 - 8 \cdot L \cdot x + 4 \cdot x^2)}{48} \cdot \frac{1}{EI}$$

$$M_D(x) := -\frac{P \cdot (L - x)}{2}$$

$$\phi_D(x) := \frac{P \cdot (L - 2 \cdot x) \cdot (3 \cdot L - 2 \cdot x)}{16} \cdot \frac{1}{EI}$$

$$V_D(x) := \frac{P}{2}$$

Funkcije uticaja u nosača:

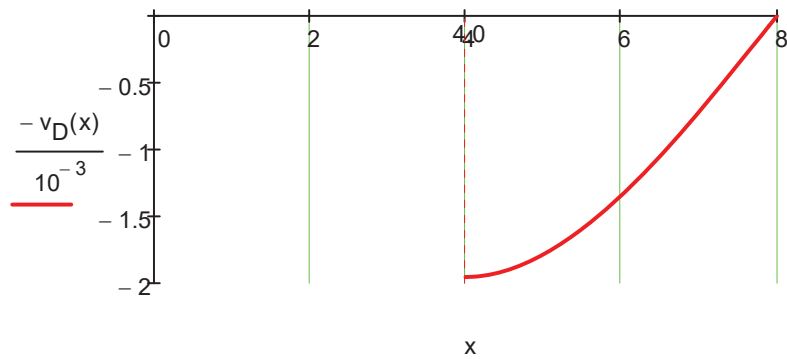
$$v(x) := \begin{cases} v_L(x) & \text{if } x \leq \frac{L}{2} \\ v_D(x) & \text{if } \frac{L}{2} \leq x \leq L \end{cases}$$

$$M(x) := \begin{cases} M_L(x) & \text{if } x \leq \frac{L}{2} \\ M_D(x) & \text{if } \frac{L}{2} \leq x \leq L \end{cases}$$

$$\phi(x) := \begin{cases} \phi_L(x) & \text{if } x \leq \frac{L}{2} \\ \phi_D(x) & \text{if } \frac{L}{2} \leq x \leq L \end{cases}$$

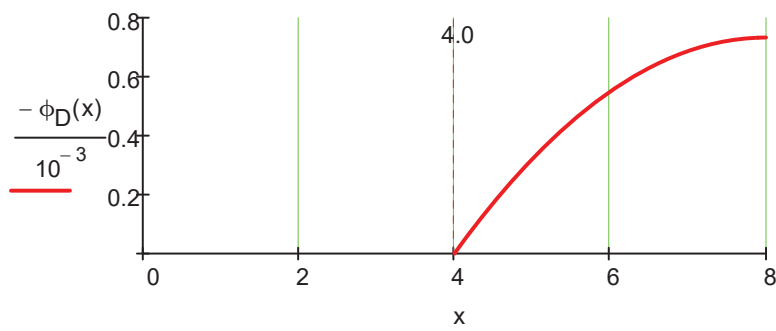
$$V(x) := \begin{cases} V_L(x) & \text{if } 0 \leq x \leq \frac{L}{2} \\ V_D(x) & \text{if } \frac{L}{2} \leq x \leq L \end{cases}$$

$x := 4, 4.1 \dots 8$



(mm)

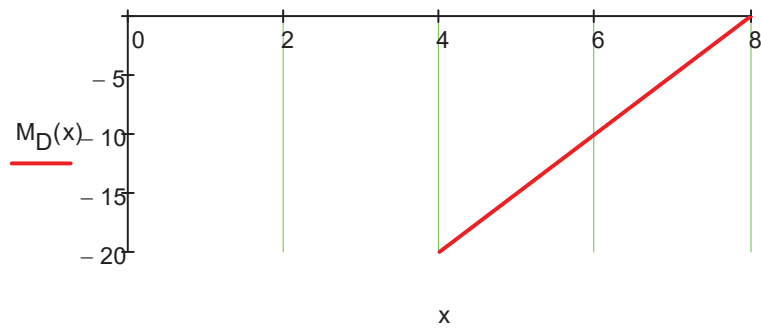
$$\frac{v_D\left(\frac{L}{2}\right)}{10^{-3}} = 1.95$$



(rad)

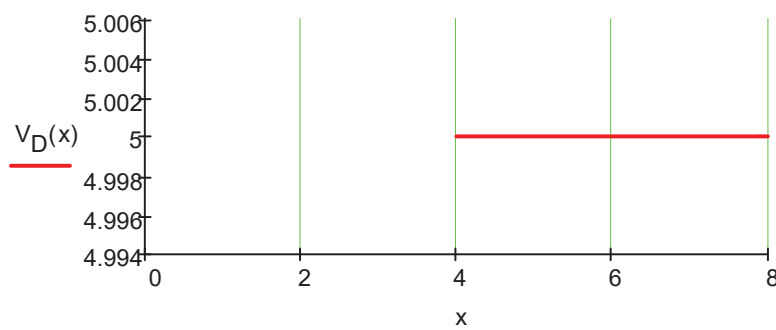
$$\phi_D\left(\frac{L}{2}\right) = 0$$

$$\phi_D(L) = -0.00073$$



(kNm)

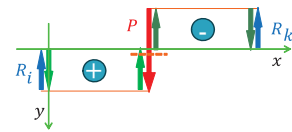
$$M_D\left(\frac{L}{2}\right) = -20$$



(kN)

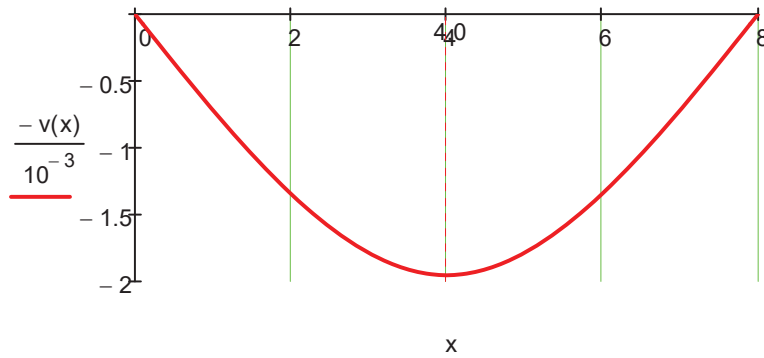
$$-V_D\left(\frac{L}{2}\right) = -5$$

$$V_D(L) = 5$$



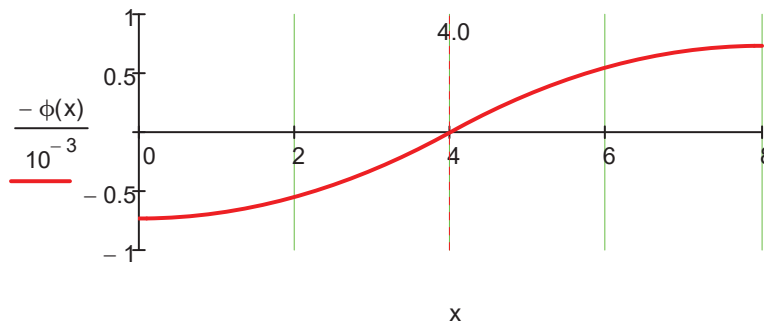
# DIJAGRAMI PRESEČNIH UTICAJA U NOSAČU

$x := 0, 0.1 \dots 8$



(mm)

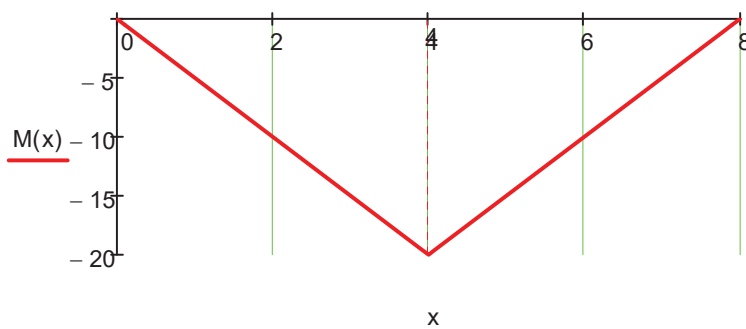
$$\frac{v\left(\frac{L}{2}\right)}{10^{-3}} = 1.95$$



(rad)

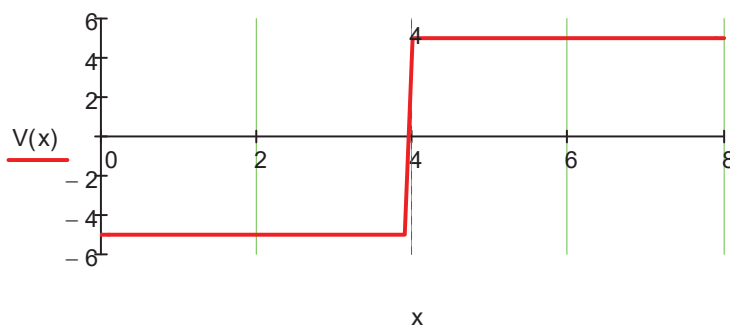
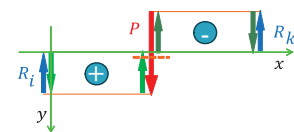
$$\phi(0) = 0.00073$$

$$\phi(L) = -0.00073$$



(kNm)

$$M\left(\frac{L}{2}\right) = -20$$



(kN)

$$-V(0) = 5$$

$$V(L) = 5$$